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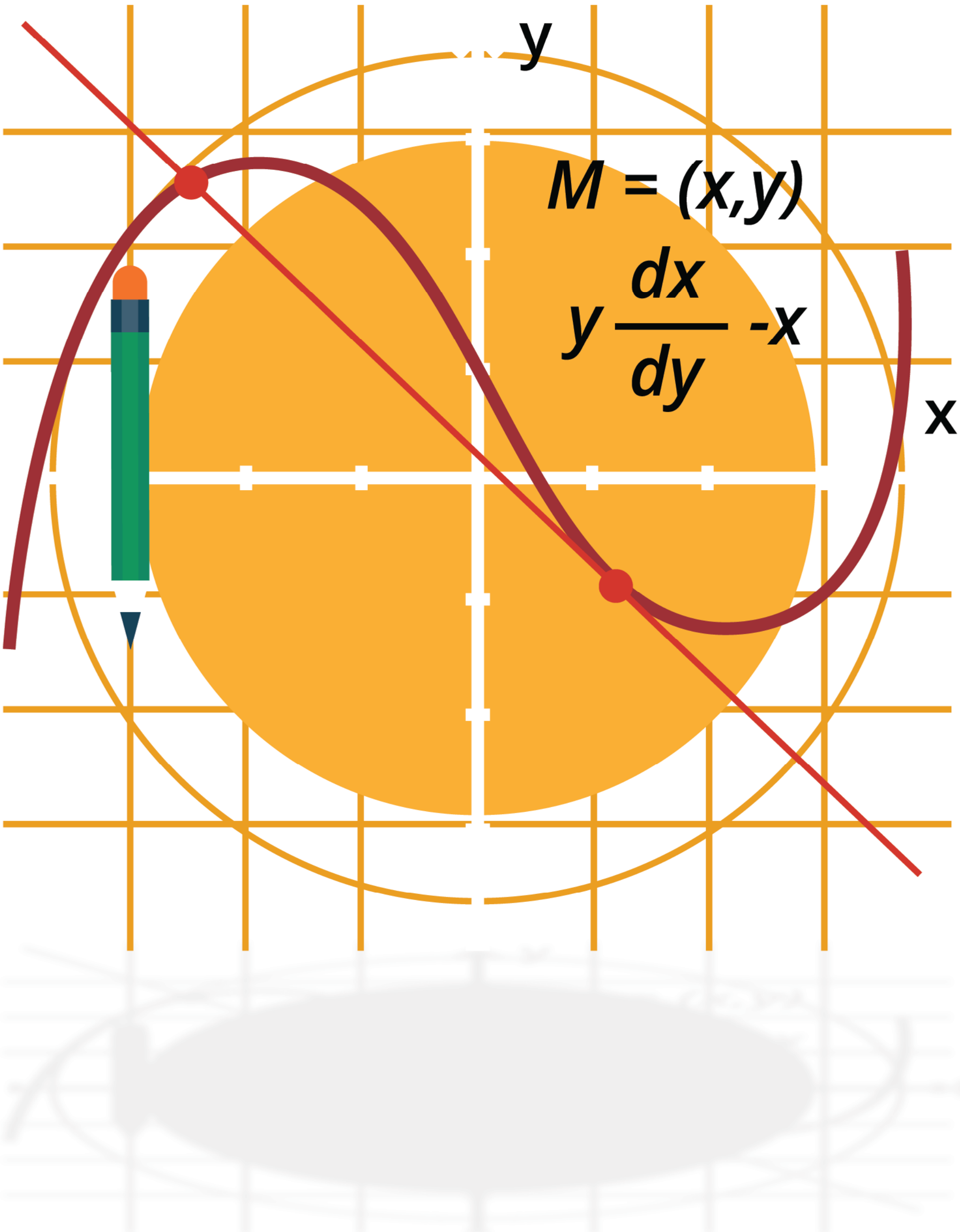
Mathematics

Class 12



05

SETS, RELATIONS & FUNCTION

SETS, RELATIONS & FUNCTION

SETS

1. SET

A set is a collection of well-defined and well distinguished objects.

1.1 Notations

The sets are usually denoted by capital letters A, B, C, etc. and the members or elements of the set are denoted by lower-case letters a, b, c, etc. If x is a member of the set A, we write $x \in A$ (read as 'x belongs to A') and if x is not a member of the set A, we write $x \notin A$ (read as 'x does not belong to A,). If x and y both belong to A, we write $x, y \in A$.

2. REPRESENTATION OF A SET

Usually, sets are represented in the following two ways :

(i) Roster form or Tabular form

(ii) Set Builder form or Rule Method

2.1 Roster Form

In this form, we list all the member of the set within braces (curly brackets) and separate these by commas. For example, the set A of all odd natural numbers less that 10 in the Roster form is written as :

$A = \{1, 3, 5, 7, 9\}$

NOTES :

(i) In roster form, every element of the set is listed only once.

(ii) The order in which the elements are listed is immaterial.

For example, each of the following sets denotes the same set $\{1, 2, 3\}$, $\{3, 2, 1\}$, $\{1, 3, 2\}$

2.2 Set-Builder Form

In this form, we write a variable (say x) representing any member of the set followed by a property satisfied by each member of the set.

For example, the set A of all prime numbers less than 10 in the set-builder form is written as

$A = \{x \mid x \text{ is a prime number less that } 10\}$

The symbol '|' stands for the words 'such that'. Sometimes, we use the symbol ':' in place of the symbol '|'.

3. TYPES OF SETS

3.1 Empty Set or Null Set

A set which has no element is called the null set or empty set. It is denoted by the symbol ϕ or { }.

For example, each of the following is a null set :

(a) The set of all real numbers whose square is –1.

(b) The set of all rational numbers whose square is 2.

(c) The set of all those integers that are both even and odd.

A set consisting of atleast one element is called a non-empty set.

3.2 Singleton Set

A set having only one element is called singleton set.

For example, {0} is a singleton set, whose only member is 0.

3.3 Finite and Infinite Set

A set which has finite number of elements is called a finite set. Otherwise, it is called an infinite set.

For example, the set of all days in a week is a finite set whereas the set of all integers, denoted by $\{...-2, -1, 0, 1, 2,...\}$ or $\{x \mid x \text{ is an integer}\}$, is an infinite set.

An empty set is a finite set.

3.4 Cardinal Number

The number of elements in finite set is represented by n(A), and is known as Cardinal number of set A.

3.5 Equal Sets

Two sets A and B are said to be equals, written as $A = B$, if every element of A is in B and every element of B is in A.

3.6 Equivalent Sets

Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$. Clearly, equal sets are equivalent but equivalent sets need not be equal.

For example, the sets $A = \{4, 5, 3, 2\}$ and $B = \{1, 6, 8, 9\}$ are equivalent but are not equal.

4. SUBSET

Let A and B be two sets. If every elements of A is an element of B, then A is called a subset of B and we write $A \subset B$ or $B \supset A$ (read as 'A is contained in B' or 'B contains A'). B is called superset of A.

NOTES :

(i) Every set is a subset and a superset of itself.

(ii) If A is not a subset of B, we write $A \not\subset B$.

(iii) The empty set is the subset of every set.

(iv) If A is a set with $n(A) = m$, then the number of subsets of A are 2^m and the number of proper subsets of A are $2^m - 1$.

For example, let $A = \{3, 4\}$, then the subsets of A are ϕ, $\{3\}$, $\{4\}$. $\{3, 4\}$. Here, $n(A) = 2$ and number of subsets of $A = 2^2 = 4$. Also, $\{3\} \subset \{3,4\}$ and $\{2,3\} \not\subset \{3, 4\}$

4.1 Power Set

The set of all subsets of a given set A is called the power set of A and is denoted by P(A).

For example, if $A = \{1, 2, 3\}$, then

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}\ \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Clearly, if A has n elements, then its power set P (A) contains exactly 2^n elements.

5. OPERATIONS ON SETS

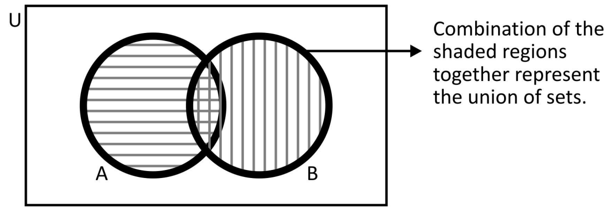
5.1 Union of Two Sets

The union of two sets A and B, written as $A \cup B$ (read as 'A union B'), is the set consisting of all the elements which are either in A or in B or in both. Thus,

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Clearly, $x \in A \cup B \Rightarrow x \in A$ or $x \in B$, and

$x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$.



For example, if $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$, then $A \cup B = \{a, b, c, d, e, f\}$

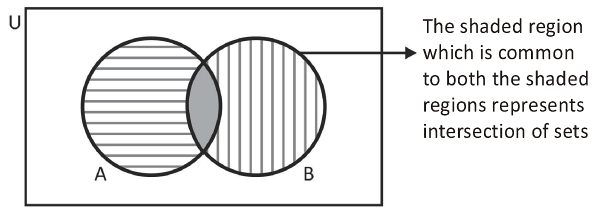
5.2 Intersection of Two sets

The intersection of two sets A and B, written as $A \cap B$ (read as 'A' intersection 'B') is the set consisting of all the common elements of A and B. Thus,

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Clearly, $x \in A \cap B \Rightarrow x \in A$ and $x \in B$, and

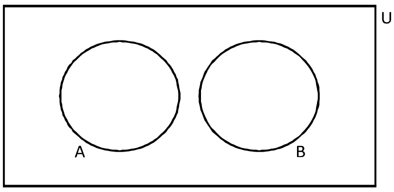
$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$.



For example, if $A = \{a, b, c, d)$ and $B = \{c, d, e, f\}$, then $A \cap B = \{c, d\}$.

5.3 Disjoint Sets

Two sets A and B are said to be disjoint, if $A \cap B = \phi$, i.e. A and B have no element in common.



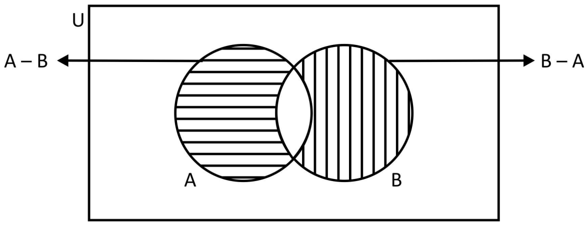
For example, if A = {1, 3, 5} and B = {2, 4, 6}, then $A \cap B = \phi$, so A and B are disjoint sets.

5.4 Difference of Two Sets

If A and B are two sets, then their difference A - B is defined as :

$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Similarly, $B - A = \{x : x \in B \text{ and } x \notin A\}$.



For example, if A = {1, 2, 3, 4, 5} and B = {1, 3, 5, 7, 9} then A - B = {2, 4} and B - A = {7, 9}.

Important Results

(a) $A - B \neq B - A$

(b) The sets A - B , B - A and $A \cap B$ are disjoint sets

(c) $A - B \subseteq A$ and $B - A \subseteq B$

(d) $A - \phi = A$ and $A - A = \phi$

5.5 Symmetric Difference of Two Sets

The symmetric difference of two sets A and B , denoted by $A \Delta B$, is defined as

$A \Delta B = (A - B) \cup (B - A)$.

For example, if A = {1,2,3,4,5} and B = {1, 3,5,7,9} then

$A \Delta B = (A - B) \cup (B - A) = \{2,4\} \cup \{7,9\} = \{2,4,7,9\}$.

5.6 Complement of a Set

If U is a universal set and A is a subset of U, then the complement of A is the set which contains those elements of U, which are not contained in A and is denoted by A'or A^c. Thus,

$A^c = \{x : x \in U \text{ and } x \notin A\}$

For example, if U = {1,2,3,4 ...} and A ={2,4,6,8,...}, then, $A^c = \{1,3,5,7, ...\}$

Important Results

(a) $U^c = \phi$ (b) $\phi^c = U$

(c) $A \cup A^c = U$ (d) $A \cap A^c = \phi$

6. ALGEBRA OF SETS

1. For any set A , we have

   (a) $A \cup A = A$ (b) $A \cap A = A$

2. For any set A, we have

   (a) $A \cup \phi = A$ (b) $A \cap \phi = \phi$

   (c) $A \cup U = U$ (d) $A \cap U = A$

3. For any two sets A and B, we have

   (a) $A \cup B = B \cup A$ (b) $A \cap B = B \cap A$

4. For any three sets A, B and C, we have

   (a) $A \cup (B \cup C) = (A \cup B) \cup C$

   (b) $A \cap (B \cap C) = (A \cap B) \cap C$

5. For any three sets A, B and C, we have

   (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

   (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. If A is any set, we have $(A^c)^c = A$.

7. DeMorgan's Laws For any two sets A and B, we have

   (a) $(A \cup B)^c = A^c \cap B^c$

(b) $(A \cap B)^c = A^c \cup B^c$

Important Results on Operations on Sets

(i) $A \subseteq A \cup B$, $B \subseteq A \cup B$, $A \cap B \subseteq A$, $A \cap B \subseteq B$

(ii) $A - B = A \cap B^c$

(iii) $(A - B) \cup B = A \cup B$

(iv) $(A - B) \cap B = \phi$ (v) $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

(vi) $A - B = B^c - A^c$

(vii) $(A \cup B) \cap (A \cup B^c) = A$

(viii) $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

(ix) $A - (A - B) = A \cap B$

(x) $A - B = B - A \Leftrightarrow A = B$

(xi) $A \cup B = A \cap B \Leftrightarrow A = B$

(xii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

7. CARDINALITY

If A, B and C are finite sets and U be the finite universal set, then

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A - B) = n(A) - n(A \cap B)$
3. $n(A \Delta B)$ = Number of elements which belong to exactly one of A or B

   $= n((A - B) \cup (B - A))$

   $= n(A - B) + n(B - A)$

   [$\because$ (A - B) and (B - A) are disjoint]

   $= n(A) - n(A \cap B) + n(B) - n(A \cap B) = n(A) + n(B) - 2n(A \cap B)$
4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
5. Number of elements in exactly two of the sets A,B,C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
6. Number of elements in exactly one of the sets A,B,C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$
7. $n(A' \cup B') = n(A \cap B)' = n(U) - n(A \cap B)$
8. $n(A' \cap B') = n(A \cup B)' = n(U) - n(A \cup B)$

RELATIONS

1. CARTESIAN PRODUCT OF SETS

Definition : Given two non-empty sets P & Q. The cartesian product $P \times Q$ is the set of all ordered pairs of elements from P & Q i.e.

$$P \times Q = \{(p, q); p \in P; q \in Q\}$$

2. RELATIONS

2.1 Definition

Let A & B be two non-empty sets. Then any subset 'R' of $A \times B$ is a relation from A to B.

If $(a, b) \in R$, then we write it as a R b which is read as a is related to b' by the relation R', 'b' is also called image of 'a' under R.

2.2 Domain and Range of a Relation

If R is a relation from A to B, then the set of first elements in R is called domain & the set of second elements in R is called range of R. symbolically.

Domain of $R = \{x : (x, y) \in R\}$

Range of $R = \{y : (x, y) \in R\}$

The set B is called co-domain of relation R.

Note that range $\subset$ co-domain.

NOTES :

Total number of relations that can be defined from a set A to a set B is the number of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and total number of relations is 2^{pq}.

2.3 Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

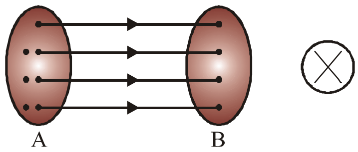
Also, Domain (R) = Range (R^{-1}) and Range (R) = Domain (R^{-1}).

3. FUNCTIONS

3.1 Definition

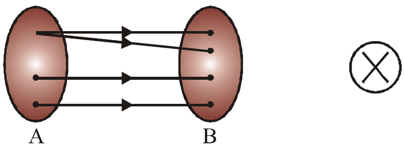
A relation 'f' from a set A to set B is said to be a function if every element of set A has one and only one image in set B.

Relations which can not be catagorized as a function



Not a function

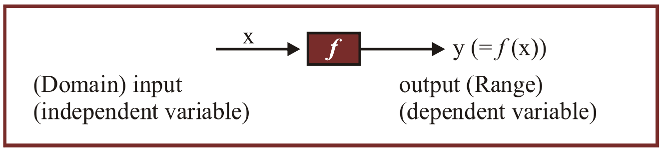
As not all elements of set A are associated with some elements of set B.

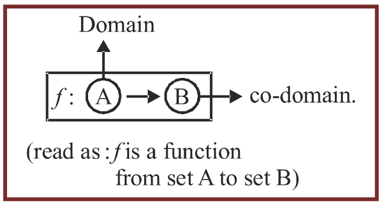


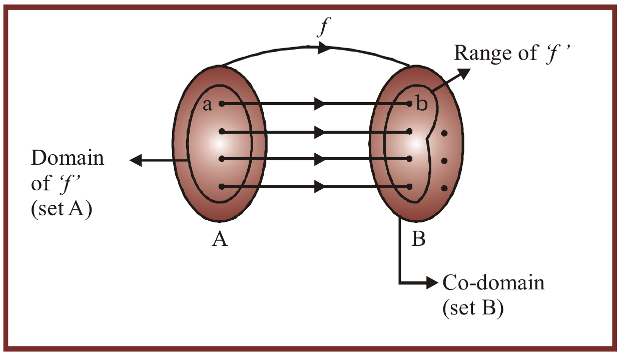
Not a function ← one-to-many

An element of set A is not associated with a unique element of set B.

Notations







3.2 Domain, Co-domain and Range of a Function

Domain : When we define $y = f(x)$ with a formula and the domain is not stated explicitly, the domain is assumed to be the largest set of x–values for which the formula gives real y–values.

The domain of $y = f(x)$ is the set of all real x for which $f(x)$ is defined (real).

Rules for finding Domain

(i) Expression under even root (i.e. square root, fourth root etc.) should be non–negative.

(ii) Denominator $\neq 0$.

(iii) $\log_a x$ is defined when $x > 0$, $a > 0$ and $a \neq 1$.

(iv) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively, then the domain of $f(x) \pm g(x)$ or $f(x) . g(x)$ is $D_1 \cap D_2$. While

domain of $\dfrac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{x: g(x) = 0\}$.

Range : The set of all f-images of elements of A is known as the range of f & denoted by $f(A)$.

Range $= f(A) = \{f(x) : x \in A\}$;

$f(A) \subseteq B$ {Range $\subseteq$ Co-domain}.

Rules for finding Range

First of all find the domain of $y = f(x)$

(i) If domain $\in$ finite number of points

$\Rightarrow$ range $\in$ set of corresponding $f(x)$ values.

(ii) If domain $\in$ R or R – {some finite points}

Put $y = f(x)$

Then express x in terms of y. From this find y for x to be defined. (i.e., find the values of y for which x exists).

(iii) If domain $\in$ a finite interval, find the least and greater value for range using monotonocity.

NOTES :

1. Question of format :

$$\left(y=\frac{Q}{Q};\ y=\frac{L}{Q};\ y=\frac{Q}{L}\right)\quad \begin{matrix} Q \to \text{quadratic} \\ L \to \text{Linear} \end{matrix}$$

Range is found out by cross-multiplying & creating a quadratic in 'x' & making $D \geq 0$ (as $x \in R$)

2. Questions to find range in which-the given expression $y = f(x)$ can be converted into x (or some function of x) = expression in 'y'.

Do this & apply method (ii).

NOTES :

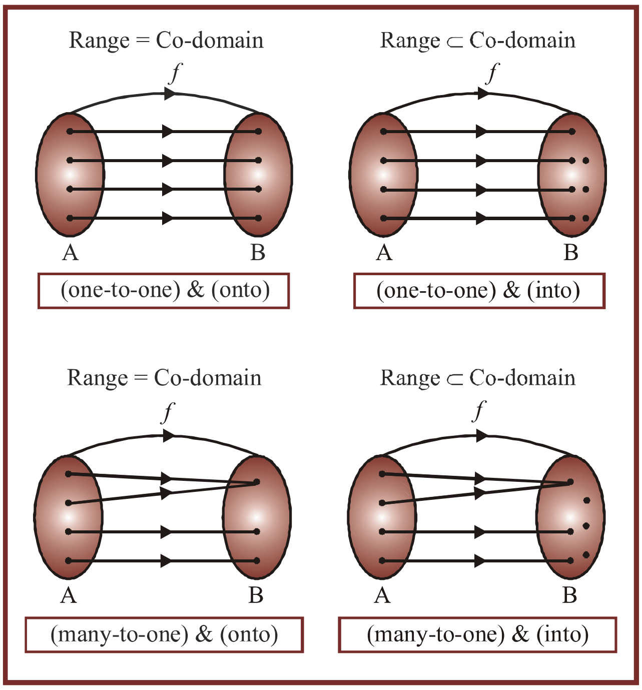
Two functions f & g are said to be equal iff

1. Domain of f = Domain of g

2. Co-domain of f = Co-domain of g

3. $f(x) = g(x)\ \forall\ x \in$ Domain.

3.3 Classification of Functions



NOTES :

(a) One-to-One functions are also called Injective functions.

(b) Onto functions are also called Surjective

(c) (one-to-one) & (onto) functions are also called Bijective Functions.

Methods to check one-one mapping

1. **Theoretically :** If $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$, then $f(x)$ is one-one.

2. **Graphically :** A function is one-one, iff no line parallel to x-axis meets the graph of function at more than one point.

3. **By Calculus :** For checking whether $f(x)$ is One-One, find whether function is only increasing or only decreasing in their domain. If yes, then function is one-one, i.e. if $f'(x) \geq 0,\ \forall\ x \in$ domain or i.e., if $f'(x) \leq 0,\ \forall\ x \in$ domain, then function is one-one.

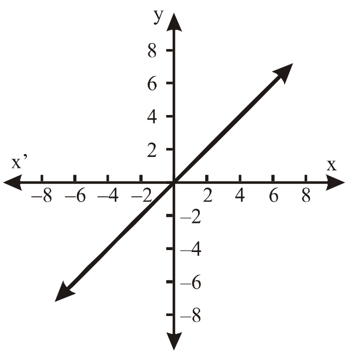
Methods to check into/onto mapping

Find the range of f(x) and compare with co-domain. If range equals co-domain then function is onto, otherwise it is into.

3.4 Some standard real functions & their graphs

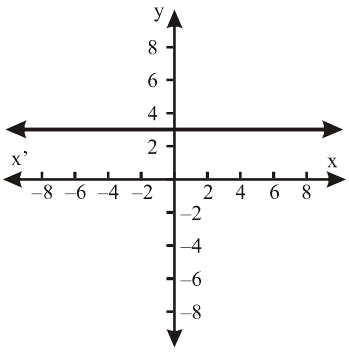
3.4.1 Identity Function : The function $f: R \to R$ defined by

$y = f(x) = x\ \forall\ x \in R$ is called identity function.



$f(x) = x$

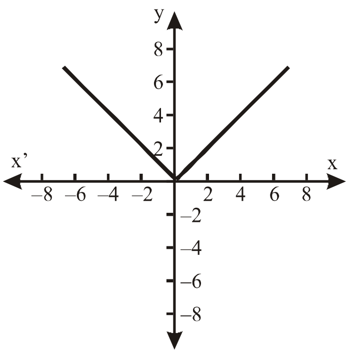
3.4.2 Constant Function : The function $f: R \to R$ defined by $y = f(x) = c, \ \forall \ x \in R$ where c is a constant is called constant function



3.4.3 Modulus Function : The function $f: R \to R$ defined by

$$f(x) = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$$

is called modulus function. It is denoted by $y = f(x) = |x|$.



Its also known as "Absolute value function'.

Properties of Modulus Function :

The modulus function has the following properties :

1. For any real number x, we have $\sqrt{x^2} = |x|$

2. $|x\,y| = |x|\,|y|$, $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

3. $\big||x| - |y|\big| \leq |x + y| \leq |x| + |y|$

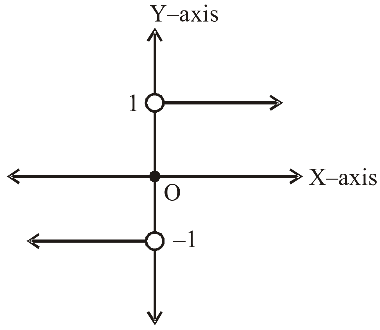
4. $\big||x| - |y|\big| \leq |x - y| \leq |x| + |y|$

(3 and 4: triangle inequality)

3.4.4 Signum Function : The function $f: R \to R$ defined by

$$f(x) = \begin{cases} 1; & x > 0 \\ 0; & x = 0 \\ -1; & x < 0 \end{cases}$$

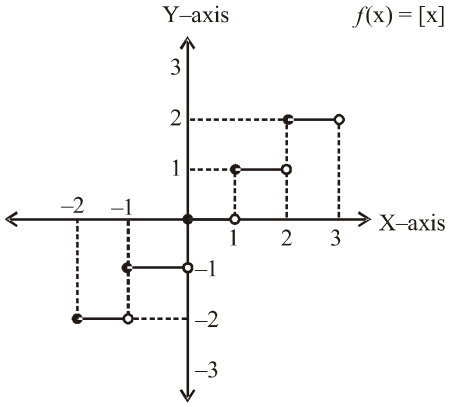
is called signum function. It is usually denoted by $y = f(x) = \text{sgn}(x)$.



NOTES :

$$\text{Sgn}(x) = \begin{cases} \frac{|x|}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$$

3.4.5 Greatest Integer Function : The function $f: R \to R$ defined as the greatest integer less than or equal to x. It is usually denoted as $y = f(x) = [x]$



Properties of Greatest Integer Function :

If n is an integer and x is any real number between n and n + 1, then the greatest integer function has the following properties :

(1) $[-n] = -[n]$

(2) $[x+n] = [x] + n$

(3) $[-x] = -[x]-1$

(4) $[x] + [-x] = \begin{cases} -1, & \text{if } x \notin I \\ 0, & \text{if } x \in I \end{cases}$

NOTES :

Fractional part of x, denoted by $\{x\}$ is given by $x-[x]$. So,

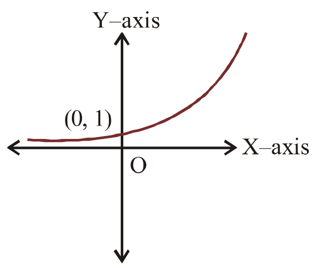
$$\{x\} = x - [x] = \begin{cases} x-1; & 1 \le x < 2 \\ x\ ; & 0 \le x < 1 \\ x+1; & -1 \le x < 0 \end{cases}$$

3.4.6 Exponential Function :

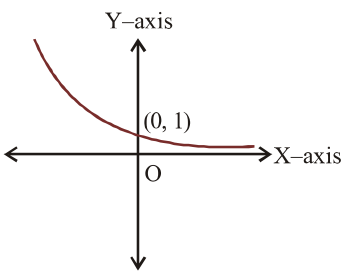
$f(x) = a^x, \quad a > 0, \quad a \neq 1$

Domain : $x \in R$

Range : $f(x) \in (0, \infty)$



$f(x) = a^x$, when $a > 1$



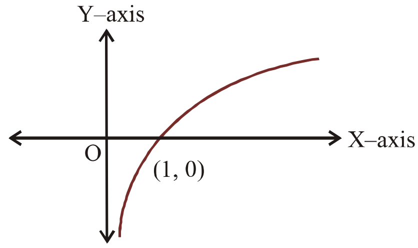
$f(x) = a^x$, when $0 < a < 1$

3.4.7 Logarithm Function :

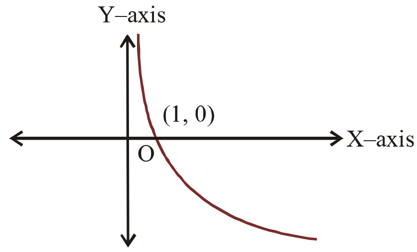
$f(x) = \log_a x, \quad a > 0, \quad a \neq 1$

Domain : $x \in (0, \infty)$

Range : $y \in R$



$f(x) = \log_a x$, when $a > 1$



$f(x) = \log_a x$, when $0 < a < 1$

(a) The Principal Properties of Logarithms

Let M & N are arbitrary positive numbers, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$.

(i) $\log_b a = c \quad \Rightarrow \quad a = b^c$

(ii) $\log_a (M . N) = \log_a M + \log_a N$

(iii) $\log_a (M/N) = \log_a M - \log_a N$

(iv) $\log_a M^N = N \log_a M$

(v) $\log_b a = \dfrac{\log_c a}{\log_c b}$, $c > 0$, $c \neq 1$.

(vi) $a^{\log_c b} = b^{\log_c a}$, $a, b, c > 0$, $c \neq 1$.

NOTES :

(a) $\log_a a = 1$

(b) $\log_b a . \log_c b . \log_a c = 1$

(c) $\log_a 1 = 0$

(d) $e^{x \ln a} = e^{\ln a^x} = a^x$

(b) Properties of Monotonocity of Logarithm

(i) If $a > 1$, $\log_a x < \log_a y \Rightarrow 0 < x < y$

(ii) If $0 < a < 1$, $\log_a x < \log_a y \Rightarrow x > y > 0$

(iii) If $a > 1$ then $\log_a x < p \Rightarrow 0 < x < a^p$

(iv) If $a > 1$ then $\log_a x > p \Rightarrow x > a^p$

(v) If $0 < a < 1$ then $\log_a x < p \Rightarrow x > a^p$

(vi) If $0 < a < 1$ then $\log_a x > p \Rightarrow 0 < x < a^p$

NOTES :

If the exponent and the base are on same side of the unity, then the logarithm is positive.

If the exponent and the base are on different sides of unity, then the logarithm is negative.

4. ALGEBRA OF REAL FUNCTION

4.1 Addition of two real functions

Let $f: X \to R$ and $g: X \to R$ by any two real functions, where $X \subset R$. Then, we define $(f+g): X \to R$ by

$(f+g)(x) = f(x) + g(x)$, for all $x \in X$.

4.2 Subtraction of a real function from another

Let $f: X \to R$ be any two any two real functions, where $X \subset R$. Then, we define $(f-g): X \to R$ by

$(f-g)(x) = f(x) - g(x)$, for all $x \in X$.

4.3 Multiplication by a scalar

Let $f: X \to R$ be a real valued function and α be a scalar. Here by scalar, we mean a real number. Then the product αf is a function from X to R defined by $(\alpha f)(x) = \alpha f(x)$, $x \in X$.

4.4 Multiplication of two real functions

The product (or multiplication) of two real functions $f: X \to R$ and $g: X \to R$ is a function $fg: X \to R$ defined by $(fg)(x) = f(x)\, g(x)$, for all $x \in X$.

This is also called *pointwise multiplication*.

4.5 Quotient of two real functions

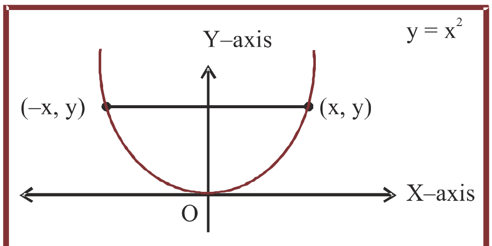
Let f and g be two real functions defined from $X \to R$ where $X \subset R$. The quotient of f by g denoted by f/g is a function defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \text{ provided } g(x) \neq 0, x \in X.$$

5. EVEN AND ODD FUNCTIONS

Even Function : $f(-x) = f(x)$, $\forall\ x \in$ Domain

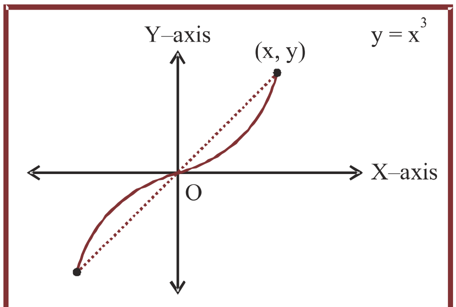
The graph of an even function $y = f(x)$ is symmetric about the y–axis. i.e., (x, y) lies on the graph $\Leftrightarrow$ (–x, y) lies on the graph.



Graph of an even function

Odd Function : $f(-x) = -f(x)$, $\forall\ x \in$ Domain

The graph of an odd function $y = f(x)$ is symmetric about origin i.e. if point (x, y) is on the graph of an odd function, then (–x, –y) will also lie on the graph.

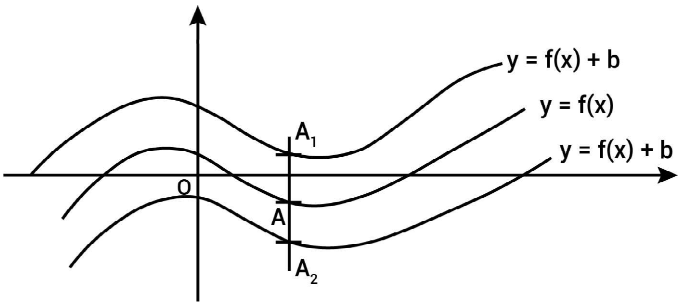


Graph of an odd function

6. GRAPHICAL TRANSFORMATION

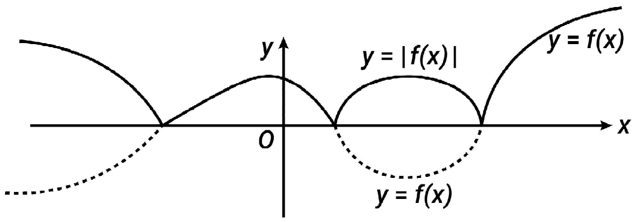
6.1 Drawing graph of $y = f(x) \pm b$, $b \in R^+$ from known graph of $y = f(x)$

It is obvious that the domain of f(x) and f(x) + b are the same. The graph of f(x) + b can be obtained by translating the graph of f(x) in the positive direction on y-axis and the graph of f(x) - b can be obtained by translating the graph of f(x) in the negative direction on y-axis.



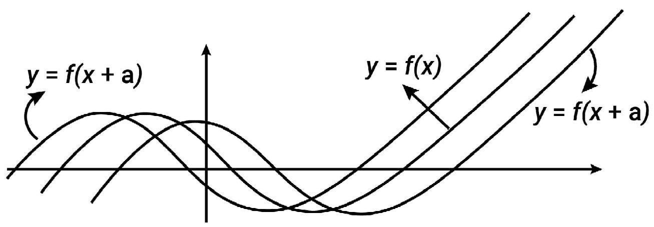
6.2 Drawing graph of $y = |f(x)|$ from known graph of $y = f(x)$

We have $|f(x)| = f(x)$ if $f(x) \geq 0$ and $|f(x)| = -f(x)$ if $f(x) < 0$ which means that the graph of f(x) and |f(x)| would concide if $f(x) \geq 0$ and the sections, where $f(x) < 0$, get inverted in the upwards direction. Figure depicts the procedure.



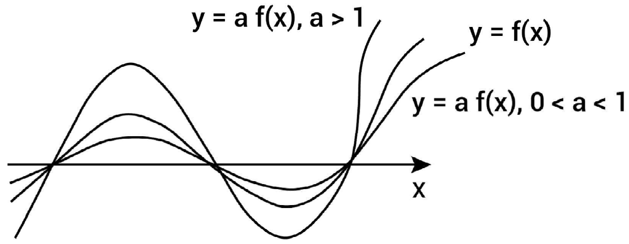
6.3 Drawing graph of $y = f(x \pm a)$, $a \in R^+$ from known graph of $y = f(x)$

The graph of f(x - a) can be obtained by translating the graph of f(x) in the positive direction on x-axis and the graph of f(x + a) can be obtained by translating the graph of f(x) in the negative direction on x-axis. The procedure is depicted in figure.



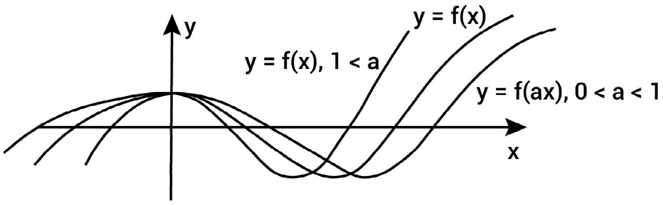
6.4 Drawing graph of $y = af(x)$, $a \in R^+$ from known graph of $y = f(x)$

We know that the corresponding points (points with the same x-coordinates) have their ordinates in the ratio of 1:a (where $a > 0$). Figure depicts the procedure.



6.5 Drawing graph of $y = f(ax)$, $a \in R^+$ from known graph of $y = f(x)$

If $0 < a < 1$, then f(x) will stretch by a times along x - axis, and if $a > 1$, then f(x) will compress by a times along x - axis. Figure depicts the procedure.



7. PERIODIC FUNCTION

Definition :

A function $f(x)$ is said to be periodic function, if there exists a positive real number T, such that $f(x+T) = f(x)$, $\forall x \in$ domain of f(x). Then, $f(x)$ is a perio dic function where least positive value of T is called fundamental period.

Graphically, if the graph repeats at fixed interval, then function is said to be periodic and its period is the width of that interval.

Some standard results on periodic functions

	Functions	Periods
(i)	$\sin^n x$, $\cos^n x$, $\sec^n x$, $\text{cosec}^n x$	π ; if n is even. 2π ; (if n is odd or fraction)
(ii)	$\tan^n x$, $\cot^n x$	π ; n is even or odd.
(iii)	$\|\sin x\|$, $\|\cos x\|$, $\|\tan x\|$ $\|\cot x\|$, $\|\sec x\|$, $\|\text{cosec } x\|$	π

(iv) $x-[x]$, [.] represents greatest integer function	1
(v) Algebraic functions e.g., $\sqrt{x}$, x^2, x^3+5, ...etc.	period does not exist

Properties of Periodic Function

(i) If $f(x)$ is periodic with period T, then

(a) $c.f(x)$ is periodic with period T.

(b) $f(x \pm c)$ is periodic with period T.

(c) $f(x) \pm c$ is periodic with period T.

where c is any constant.

(ii) If $f(x)$ is periodic with period T, then

$kf(cx+d)$ has period $T/|c|$,

i.e. Period is only affected by coefficient of x where k, c, d are constants.

(iii) If $f_1(x), f_2(x)$ are periodic functions with periods T_1, T_2 respectively, then $h(x) = af_1(x) \pm bf_2(x)$ has period as, LCM of $\{T_1, T_2\}$

NOTES :

(a) $\text{LCM of}\left(\frac{a}{b}, \frac{c}{d}, \frac{e}{f}\right) = \frac{\text{LCM of }(a, c, e)}{\text{HCF of }(b, d, f)}$

(b) LCM of rational and rational always exists.

LCM of irrational and irrational sometime exists.

But LCM of rational and irrational never exists.

e.g., LCM of $(2\pi, 1, 6\pi)$ is not possible as

$2\pi, 6\pi \in$ irrational and $1 \in$ rational.

SOLVED EXAMPLES

Example – 1

Write the set of all positive integers whose cube is odd.

Sol. The elements of the required set are not even.

[$\because$ Cube of an even integer is also an even integer]

Moreover, the cube of a positive odd integer is a positive odd integer.

$\Rightarrow$ The elements of the required set are all positive odd integers.

Hence, the required set, in the set builder form, is :

$\{2k+1 : k \geq 0, k \in Z\}$.

Example – 2

Write the set $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}\right\}$ in the set builder form.

Sol. In each element of the given set the denominator is one more than the numerator.

Also the numerators are from 1 to 7.

Hence the set builder form of the given set is :

$\{x : x = n/n+1, n \in N \text{ and } 1 \leq n \leq 7\}$.

Example – 3

Write the set $\{x : x$ is a positive integer and $x^2 < 30\}$ in the roster form.

Sol. The squares of positive integers whose squares are less than 30 are : 1, 2, 3, 4, 5.

Hence the given set, in roster form, is $\{1, 2, 3, 4, 5\}$.

Example – 4

Write the set $\{0, 1, 4, 9, 16, ...\}$ in set builder form.

Sol. The elements of the given set are squares of integers :

$0, \pm 1, \pm 2, \pm 3, \pm 4, ...$

Hence the given set, in set builder form, is $\{x^2 : x \in Z\}$.

Example – 5

State which of the following sets are finite and which are infinite

(i) $A = \{x : x \in N \text{ and } x^2 - 3x + 2 = 0\}$

(ii) $B = \{x : x \in N \text{ and } x^2 = 9\}$

(iii) $C = \{x : x \in N$ and x is even$\}$

(iv) $D = \{x : x \in N \text{ and } 2x - 3 = 0\}$.

Sol. (i) $A = \{1, 2\}$.

[$\because x^2 - 3x + 2 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, 2$]

Hence A is finite.

(ii) $B = \{3\}$.

[$\because x^2 = 9 \Rightarrow x = \pm 3$. But $3 \in N$]

Hence B is finite.

(iii) $C = \{2, 4, 6, ...\}$

Hence C is infinite.

(iv) $D = \phi$. $\left[\because 2x - 3 = 0 \Rightarrow x = \frac{3}{2} \notin N\right]$

Hence D is finite.

Example – 6

Which of the following are empty (null) sets ?

(i) Set of odd natural numbers divisible by 2

(ii) $\{x : 3 < x < 4, x \in N\}$

(iii) $\{x : x^2 = 25$ and x is an odd integer$\}$

(iv) $[x : x^2 - 2 = 0$ and x is rational$]$

(v) $\{x : x$ is common point of any two parallel lines$\}$.

Sol. (i) Since there is no odd natural number, which is divisible by 2.

$\therefore$ it is an empty set.

(ii) Since there is no natural number between 3 and 4.

$\therefore$ it is an empty set.

(iii) Now $x^2 = 25 \Rightarrow x = \pm 5$, both are odd.

$\therefore$ The set $\{-5, 5\}$ is non-empty.

(iv) Since there is no rational number whose square is 2,

$\therefore$ the given set is an empty set.

(v) Since any two parallel lines have no common point,

$\therefore$ the given set is an empty set.

Example – 7

Find the pairs of equal sets from the following sets, if any, giving reasons :

$A = \{0\}, B = \{x : x > 15 \text{ and } x < 5\}$,

$C = \{x : x - 5 = 0\}, D = \{x : x^2 = 25\}$,

E = {x : x is a positive integral root of the equation $x^2 - 2x - 15 = 0\}$.

Sol. Here we have,

$A = \{0\}$

$B = \phi$

[$\because$ There is no number, which is greater than 15 and less than 5]

$C = \{5\}$ [$\because x - 5 = 0 \Rightarrow x = 5$]

$D = \{-5, 5\}$ [$\because x^2 = 25 \Rightarrow x = \pm 5$]

and $E = \{5\}$.

[$\because x^2 - 2x - 15 = 0 \Rightarrow (x-5)(x+3) = 0 \Rightarrow x = 5, -3$. Out of these two,

5 is positive integral]

Clearly $C = E$.

Example – 8

Are the following pairs of sets equal ? Give reasons.

(i) $A = \{1, 2\}, B = \{x : x \text{ is a solution of } x^2 + 3x + 2 = 0\}$

(ii) A = {x : x is a letter in the word FOLLOW},

B = {y : y is a letter in the word WOLF}.

Sol. (i) $A = \{1, 2\}, B = \{-2, -1\}$

[$\because x^2 + 3x + 2 = 0 \Rightarrow (x+2)(x+1) = 0 \Rightarrow x = -2, -1$]

Clearly $A \neq B$.

(ii) A = {F, O, L, L, O, W} = {F, O, L, W}

B = {W, O, L, F} = {F, O, L, W}.

Clearly A = B.

Example – 9

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, A = \{1, 2, 3, 4\}, B = \{2, 4, 6, 8\}$. Find :

(i) A^C (ii) B^C (iii) $(A^C)^C$ (iv) $(A \cup B)^C$

Sol. (i) A^C = Set of those elements of U, which are not in $A = \{5, 6, 7, 8, 9\}$.

(ii) B^C = Set of those elements of U, which are not in $B = \{1, 3, 5, 7, 9\}$.

(iii) $(A^C)^C$ = Set of those elements of U, which are not in $A' = \{1, 2, 3, 4\} = A$.

(iv) $A \cup B = \{1,2,3,4\} \cup \{2,4,6,8\} = \{1,2,3,4,6,8\}$.

$\therefore$ $(A \cup B)^C$ = Set of those elements of U, which are not in $(A \cup B) = \{5, 7, 9\}$.

Example – 10

If U = {x : x is a letter in English alphabet}, A = {x : x is a vowel in English alphabet}.

Find A^C and $(A^C)^C$.

Sol. (i) Since A = {x : x is a letter in English alphabet},

$\therefore$ A^C is the set of those elements of U, which are not vowels

= {x : x is a consonant in English alphabet}.

(ii) $(A^C)^C$ is the set of those elements of U, which are not consonants = {x : x is a vowel in English alphabet} = A.

Hence $(A^C)^C = A$.

Example – 11

Let $A = \{1, 2, 3, 4, 5\}, B = \{3, 4, 5, 6, 7\}, C = \{6, 7, 8, 9\}$ and $D = \{7, 8, 9, 10\}$. Find :

(a) (i) $A \cup B$ (ii) $B \cup D$

(iii) $A \cup B \cup C$ (iv) $B \cup C \cup D$

(b) (i) $A \cap B$ (ii) $B \cap D$ (iii) $A \cap B \cap C$.

Sol. (a) (i) $A \cup B = \{1, 2, 3, 4, 5\} \cup \{3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$.

(ii) $B \cup D = \{3, 4, 5, 6, 7\} \cup \{7, 8, 9, 10\}$

$= \{3, 4, 5, 6, 7, 8, 9, 10\}$.

(iii) $A \cup B \cup C = \{1,2,3,4,5\} \cup \{3,4,5,6,7\} \cup \{6,7,8,9\}$.

$= \{1,2,3,4,5,6,7\} \cup \{6,7,8,9\} = \{1,2,3,4,5,6,7,8,9\}$.

(iv) $B \cup C \cup D = \{3,4,5,6,7\} \cup \{6,7,8,9\} \cup \{7,8,9,10\}$.

$= \{3,4,5,6,7,8,9\} \cup \{7,8,9,10\} = \{3,4,5,6,7,8,9,10\}$.

(b) (i) $A \cap B = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} = \{3, 4, 5\}$.

(ii) $B \cap D = \{3, 4, 5, 6, 7\} \cap \{7, 8, 9, 10\} = \{7\}$.

(iii) $A \cap B \cap C = \{1, 2, 3, 4, 5\} \cap \{3, 4, 5, 6, 7\} \cap \{6, 7, 8, 9\} = \{3, 4, 5\} \cap \{6, 7, 8, 9\} = \phi$.

Example – 12

If $A_1 = \{2, 3, 4, 5\}$, $A_2 = \{3, 4, 5, 6\}$, $A_3 = \{4, 5, 6, 7\}$, find $\cup A_i$ and $\cap A_i$, where $i = \{1, 2, 3\}$.

Sol. (i) $\cup A_i = A_1 \cup A_2 \cup A_3 = \{2,3,4,5\} \cup \{3,4,5,6\} \cup \{4,5,6,7\}$

$= \{2,3,4,5\} \cup \{3,4,5,6,7\} = \{2,3,4,5,6,7\}$.

(ii) $\cap A_i = A_1 \cap A_2 \cap A_3 = \{2,3,4,5\} \cap \{3,4,5,6\} \cap \{4,5,6,7\}$

$= \{2,3,4,5\} \cap \{4,5,6\} = \{4,5\}$.

Example – 13

Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$. Find $(A-B) \cup (B-A)$.

Sol. We have, $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{3, 4, 5, 6, 7, 8\}$.

$\therefore A-B = \{1, 2\}$ and $B-A = \{7, 8\}$

$\therefore (A-B) \cup (B-A) = \{1, 2\} \cup \{7, 8\} = \{1, 2, 7, 8\}$.

Example – 14

Prove that :

$A \cap (B-C) = (A \cap B) - (A \cap C)$

Sol. Let x be an arbitrary element of $A \cap (B-C)$.

Then $x \in A \cap (B-C)$

$\Rightarrow$ $x \in A$ and $x \in (B-C)$

$\Rightarrow$ $x \in A$ and ($x \in B$ and $x \notin C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) and ($x \in A$ and $x \notin C$)

$\Rightarrow$ $x \in (A \cap B)$ and $x \notin (A \cap C)$

$\Rightarrow$ $x \in \{(A \cap B) - (A \cap C)\}$

$\therefore$ $A \cap (B-C) \subseteq (A \cap B) - (A \cap C)$ ... (1)

Let y be an arbitrary element of $(A \cap B) - (A \cap C)$.

Then $y \in (A \cap B) - (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ and $y \notin (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) and ($y \in A$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ and $y \notin C$)

$\Rightarrow$ $y \in A$ and $y \in (B-C)$

$\Rightarrow$ $y \in A \cap (B-C)$

$\therefore$ $(A \cap B) - (A \cap C) \subseteq A \cap (B-C)$ ... (2)

Combining (1) and (2).

$A \cap (B-C) = (A \cap B) - (A \cap C)$.

Example – 15

Prove the following :

$A \subset B \Leftrightarrow B^c \subset A^c$

Sol. Let $x \in B^c$, where x is arbitrary.

Now $x \in B^c$

$\Rightarrow$ $x \notin B$

$\Rightarrow$ $x \notin A$ [$\because A \subset B$]

$\Rightarrow$ $x \in A^c$

$\therefore$ $B^c \subset A^c$ ... (1)

Conversely : Let $x \in A$, where x is arbitrary.

Now $x \in A$

$\Rightarrow$ $x \notin A^c$

$\Rightarrow$ $x \notin B_c$ [$\because B^c \subset A^c$]

$\Rightarrow$ $x \in B$

$\therefore$ $A \subset B$

Combining (1) and (2), $A \subset B \Leftrightarrow B^c \subset A^c$.

Example – 16

Prove the following :

$A - B = A - (A \cap B)$

where U is the universal set.

Sol. Let $x \in (A-B)$, where x is arbitrary.

Now $x \in (A-B)$

$\Leftrightarrow$ $x \in A$ and $x \notin B$

$\Leftrightarrow$ ($x \in A$ and $x \in A$) and $x \notin B$

[Note this step]

$\Leftrightarrow$ $x \in A$ and ($x \in A$ and $x \notin B$)

[Associative Law]

$\Leftrightarrow$ $x \in A$ and $x \notin (A \cap B)$

$\Leftrightarrow$ $x \in A - (A \cap B)$

Hence $A - B \subset A - (A \cap B)$.

Now Let $y \in A - (A \cap B)$

$\Rightarrow y \in A$ and $y \notin (A \cap B)$

$\Rightarrow y \in A$ and $y \notin B$

$\Rightarrow y \in A - B$.

So, $A - B = A - (A \cap B)$.

Example – 17

If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then

(a) $A = C$ (b) $B = C$

(c) $A \cap B = \phi$ (d) $A = B$

Ans. (b)

Sol. Let $x \in C$

Suppose $x \in A \Rightarrow x \in A \cap C$

$\Rightarrow x \in A \cap B$ $(\because A \cap C = A \cap B)$

Thus $x \in B$

Again suppose $x \notin A \Rightarrow x \in C \cup A$

$\Rightarrow x \in B \cup A \Rightarrow x \in B$

Thus in both cases $x \in C \Rightarrow x \in B$

Hence $C \subseteq B$ ..(i)

Similarly we can show that $B \subseteq C$ ...(ii)

Combining (i) and (ii) we get $B = C$.

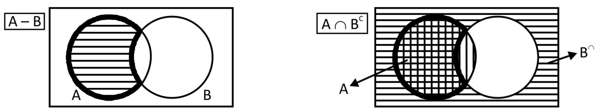
Example – 18

If A and B are any two sets, prove using Venn Diagrams

(i) $A - B = A \cap B^C$ (ii) $(A - B) \cup B = A \cup B$.

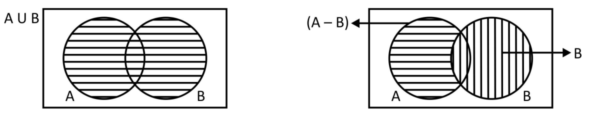
Sol.

(i)



Shaded region in both are the same. The common shaded area represents the intersection.

(ii)



Combination of all the shaded regions represents the union.

Example – 19

If $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ and $C = \{7, 8, 9\}$, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol. We have, $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$ and $C = \{7, 8, 9\}$.

$\therefore$ $A \cup B = \{1, 2, 3\} \cup \{4, 5, 6\} = \{1, 2, 3, 4, 5, 6\}$ ...(1)

$A \cup C = \{1, 2, 3\} \cup \{7, 8, 9\}$

$= \{1, 2, 3, 7, 8, 9\}$ ...(2)

and $B \cap C = \{4, 5, 6\} \cap \{7, 8, 9\} = \phi$ ...(3)

Now $A \cup (B \cap C) = \{1, 2, 3\} \cup \phi = \{1, 2, 3\}$ ...(4)

and $(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6\} \cap \{1, 2, 3, 7, 8, 9\}$

$= \{1, 2, 3\}$ ...(5)

From (4) and (5), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, which verifies the result.

Example – 20

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(i) $(A \cup B)^C = A^C \cap B^C$ (ii) $(A \cap B)^C = A^C \cup B^C$.

Sol. We have, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$.

(i) $A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$

$\therefore$ $= \{2, 3, 4, 5, 6, 7, 8\}$

$\therefore$ $(A \cup B)^C = \{1, 9\}$ ...(1)

Also $A^C = \{1, 3, 5, 7, 9\}$

and $B^C = \{1, 4, 6, 8, 9\}$

$\therefore$ $A^C \cap B^C = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$

$= \{1, 9\}$ ...(2)

From (1) and (2), $(A \cup B)^C = A^C \cap B^C$, which verifies the result.

(ii) $A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

$\therefore$ $(A \cap B)^C = \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(3)

and $A^C \cup B^C = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$

$= \{1, 3, 4, 5, 6, 7, 8, 9\}$ ...(4)

From (3) and (4), $(A \cap B)^C = A^C \cup B^C$, which verifies the result.

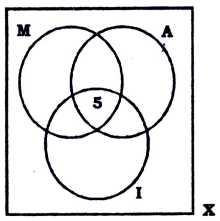
Example – 21

In a class of 200 students who appeared in a certain examination. 35 students failed in MHTCET, 40 in AIEEE, 40 in IIT, 20 failed in MHTCET and AIEEE, 17 in AIEEE and IIT, 15 in MHTCET and IIT and 5 failed in all three examinations. Find how many students

(i) Did not fail in any examination.

(ii) Failed in AIEEE or IIT.

Sol.



$n(M) = 35, n(A) = 40, n(I) = 40$

$n(M \cap A) = 20, n(A \cap I) = 17,$

$n(I \cap M) = 15, n(M \cap A \cap I) = 5$

$n(X) = 200$

$n(M \cup A \cup I) = n(M) + n(A) + n(I) -$

$n(M \cap A) - n(A \cap I) - n(M \cap I) + n(M \cap A \cap I)$

$= 35 + 40 + 40 - 20 - 17 - 15 + 5 = 68$

(i) Number of students passed in all three examination

$= 200 - 68 = 132$

(ii) Number of students failed in IIT or AIEEE

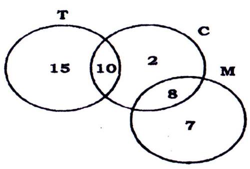
$= n(I \cup A) = n(I) + n(A) - n(I \cap A)$

$= 40 + 40 - 17 = 63$

Example – 22

In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of the them take tea and milk both and everyone takes atleast one beverage, find the number of students in the hostel.

Sol.



Let the sets, T, C and M are the students who drink tea, coffee and milk respectively. This problem can be solved by Venn diagram.

$n(T) = 25; n(C) = 20; n(M) = 15$

$n(T \cap C) = 10; n(M \cap C) = 8$

Number of students in hostel

$= n(T \cup C \cup M)$

$\therefore n(T \cup C \cup M) = 15 + 10 + 2 + 8 + 7 = 42$

Example – 23

If $A = \{1, 2\}$, find $A \times A \times A$

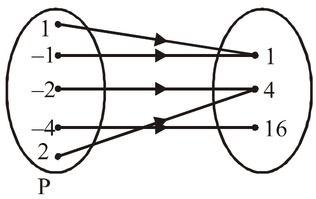
Sol. $A \times A \times A = \{(x, y, z), x \in A, y \in A, z \in A\}$

First find A×A than find A×A×A

so, $A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (2, 2, 2), (2, 2, 1), (2, 1, 2), (1, 2, 2)\}$

Example – 24

Following figure shows a relation between sets P and Q. Write this relation in (i) set builder form, (ii) roster form



Sol. It is clear, that relation R is "y is the square of x".

(i) In set builder form, $R = \{(x, y) : y = x^2, x \in P, y \in Q\}$

(ii) In roster form,

$R = \{(1, 1), (-1, 1), (2, 4), (-2, 4) (-4, 16)\}$

Example – 25

Let R be the relation on Z defined by R = {(a, b); a, b, ∈ Z, a – b is an integer}. Find domain and range of R.

Sol. As for any two integers a & b, a – b is an integer hence domain and range is all integers.

Example – 26

Determine domain and range of :-

$$R = \left\{\left(x+4, \frac{2+x}{2-x}\right) : 4 \le x \le 6,\ x \in N\right\}$$

Sol. $R = \left\{(8, -3), \left(9, -\frac{7}{3}\right), (10, -2)\right\}$

By taking $x = 4, 5, 6$

so, domain = {8, 9, 10}

range = $\left\{-3, -\frac{7}{3}, -2\right\}$

Example – 27

Let A = {1, 2}. List all the relations on A.

Sol. Given A = {1, 2}

$A \times A = \{(1,1), (1,2), (2,1), (2,2)\}$

Since a relation R from set A to set A is a subset of A × A

∴ All the relations on A are :

ϕ, {(1, 1)}, {(1, 2)}, {(2, 1)}, {(2, 2)}, {(1, 1), (1, 2)}, {(1, 1), (2, 1)}, {(1, 1), (2, 2)}, {(1, 2), (2, 1)}, {(1, 2), (2, 2)}, {(2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1)}, {(1, 1), (1, 2), (2, 2)}, {(1, 1), (2, 1), (2, 2)}, {(1, 2), (2, 1), (2, 2)}, {(1, 1), (1, 2), (2, 1), (2, 2)}.

Since n(A × A) = 4, the number of all relations on the set A = 2^4 i.e., 16.

(As number of subsets of a set with n elements is 2^n)

Example – 28

The solution set of $x^2 + 2 \le 3x \le 2x^2 - 5$ is

(a) ϕ (b) [1, 2]

(c) $(-\infty, -1) \cup [5/2, \infty)$ (d) none

Ans. (a)

Sol. $x^2 + 2 \le 3x \le 2x^2 - 5$

$x^2 + 2 \le 3x$ *and* $3x \le 2x^2 - 5$

$x^2 - 3x + 2 \le 0$ *and* $2x^2 - 3x - 5 \ge 0$

$(x-1)(x-2) \le 0$ *and* $(2x-5)(x+1) \ge 0$

$\Rightarrow x \in [1,2]$ *and* $x \in (-\infty, -1] \cup \left[\frac{5}{2}, \infty\right)$

$\Rightarrow x \in \phi$

Example – 29

Find the set of values of 'x' for which the given conditions are true :

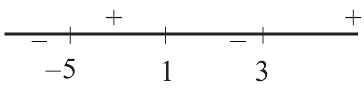
(a) $-(x-1)(x-3)(x+5) < 0$

(b) $\frac{(x-1)(x-2)}{(x-3)} \le 0$

Ans. (a) $(-5, 1) \cup (3, \infty)$ (b) $(-\infty, 1] \cup [2, 3)$

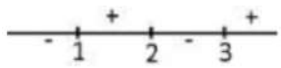
Sol. (a) $-(x-1)(x-3)(x+5) < 0$

$\Rightarrow (x-1)(x-3)(x+5) > 0$



$\Rightarrow x \in (-5, 1) \cup (3, \infty)$

(b) $\frac{(x-1)(x-2)}{(x-3)} \le 0$



$x \in (-\infty, 1] \cup [2, 3)$

Example – 30

The number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is

(a) 4 (b) 5

(c) 3 (d) none of these

Ans. (c)

Sol. $\frac{x+2}{x^2+1} > \frac{1}{2}$

$2x + 4 > x^2 + 1 \quad (\because x^2 + 1 > 0)$

$x^2 - 2x - 3 < 0$

$\Rightarrow (x-3)(x+1) < 0$

$\Rightarrow x \in (-1, 3)$

Number of integer values = 3

Example – 31

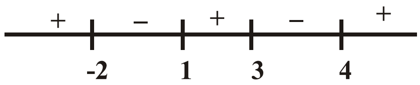
Find the domain of definition of the following

function : $f(x)=\sqrt{\dfrac{(x-1)(x+2)}{(x-3)(x-4)}}$

Sol. For $f(x)$ to be defined $\dfrac{(x-1)(x+2)}{(x-3)(x-4)} \geq 0$ and $x \neq 3, 4$

By wavy – curve method the domain of definition of $f(x)$ is the set

$x \in (-\infty, -2] \cup [1, 3) \cup (4, \infty)$.



Example – 32

Find domain for $f(x)=\sqrt{\cos(\sin x)}$.

Sol. $f(x)=\sqrt{\cos(\sin x)}$ is defined, if

$\cos(\sin x) \geq 0$

As, we know

$-1 \leq \sin x \leq 1$ for all x

$\cos\theta \geq 0$

(Here, $\theta = \sin x$ lies in the 1st and 4th quadrants)

i.e. $\cos(\sin x) \geq 0$, for all x

i.e. $x \in R$.

Thus, domain $f(x) \in R$

Example – 33

A function f is defined on the set $\{1, 2, 3, 4, 5\}$ as follows :

$$f(x)=\begin{cases} 1+x & \text{if } 1 \leq x < 2 \\ 2x-1 & \text{if } 2 \leq x < 4 \\ 3x-10 & \text{if } 4 \leq x < 6 \end{cases}$$

(i) Find the domain of the function.

(ii) Find the range of the function.

(iii) Find the values of $f(2), f(3), f(4), f(6)$.

Sol. (i) Domain : $\{1, 2, 3, 4, 5\}$

(ii) Range :

$f(1)=1+1=2$ $\quad f(4)=3(4)-10=2$

$f(2)=2(2)-1=3$ $\quad f(5)=3(5)-10=5$

$f(3)=2(3)-1=5$

So, range is $\{2, 3, 5\}$

(iii) $f(2)=3, f(4)=2, f(3)=5, f(6)$ is not defined as 6 is not in domain.

Example – 34

Let $A=\{1, 2\}$. List all the relations on A.

Sol. Given $A=\{1, 2\}$

$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

Since a relation R from set A to set A is a subset of $A \times A$

$\therefore$ All the relations on A are :

ϕ, $\{(1, 1)\}$, $\{(1, 2)\}$, $\{(2, 1)\}$, $\{(2, 2)\}$, $\{(1, 1), (1, 2)\}$, $\{(1, 1), (2, 1)\}$, $\{(1, 1), (2, 2)\}$, $\{(1, 2), (2, 1)\}$, $\{(1, 2), (2, 2)\}$, $\{(2, 1), (2, 2)\}$, $\{(1, 1), (1, 2), (2, 1)\}$, $\{(1, 1), (1, 2), (2, 2)\}$, $\{(1, 1), (2, 1), (2, 2)\}$, $\{(1, 2), (2, 1), (2, 2)\}$, $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$.

Since $n(A \times A) = 4$, the number of all relations on the set $A = 2^4$ i.e., 16.

(As number of subsets of a set with n elements is 2^n)

Example – 35

Find the domain and range of the following functions

(i) $\left\{\left(x, \dfrac{x^2-1}{x-1}\right) : x \in R, x \neq 1\right\}$

(ii) $\left\{\left(x, \dfrac{1}{1-x^2}\right) : x \in R, x \neq \pm 1\right\}$

Sol. (i) Let $f(x)=\left\{\left(x, \dfrac{x^2-1}{x-1}\right) : x \in R, x \neq 1\right\}$

Clearly, f is not defined when $x = 1$

$\therefore$ f is defined for all real values of x except $x = 1$

$\therefore$ Domain $= R - \{1\}$

Let $y=\dfrac{x^2-1}{x-1}=x+1 \,(\text{as } x \neq 1)$

$\therefore$ $x = y - 1$

Clearly $y \neq 2$ as $x \neq 1$

$\therefore$ Range $= R - \{2\}$.

(ii) Let $f(x)=\left\{\left(x, \frac{1}{1-x^2}\right): x \in R,\ x = \pm 1\right\}$

Clearly, $f(x)=\frac{1}{1-x^2}$ is not defined when $1-x^2=0$

i.e., when $x=\pm 1$

$\therefore$ Domain $= R-\{1,-1\}$

Further, $y=\frac{1}{1-x^2}$ Since $x \neq \pm 1$

$\Rightarrow \left(1-x^2\right)=\frac{1}{y} \quad \Rightarrow \quad x=\pm\sqrt{\left(1-\frac{1}{y}\right)}=\pm\sqrt{\frac{y-1}{y}}$

$\therefore$ x is defined when $y \in (-\infty, 0) \cup [1, \infty)$.

$\Rightarrow$ $y \in (-\infty, 0) \cup [1, \infty)$

$\therefore$ Range $= (-\infty, 0) \cup [1, \infty)$.

Example – 36

Find the range of the following function :

$$f(x)=ln\sqrt{x^2+4x+5}$$

Sol. Here $f(x)=\ln\sqrt{x^2+4x+5}=\ln\sqrt{(x+2)^2+1}$

i.e. x^2+4x+5 takes all values in $[1, \infty)$

$\Rightarrow$ $f(x)$ will take all values in $[0, \infty)$.

Hence range of $f(x)$ is $[0, \infty)$.

Example – 37

Find the range of the function $y=\frac{1}{2-\sin 3x}$

Sol. Clearly, as Denominator $(2-\sin 3x) \neq 0$

$\Rightarrow$ Domain : $x \in R$

We have, $y=\frac{1}{2-\sin 3x}$

Note : (sin 3x) can be seperated & written as a function of y

$\Rightarrow$ $2-\sin 3x=\frac{1}{y}$

$\Rightarrow$ $\sin 3x=\frac{2y-1}{y}$

for x to be real

$\Rightarrow$ $-1 \le \frac{2y-1}{y} \le 1$ (since, $-1 \le \sin 3x \le 1$)

$-1 \le \frac{2y-1}{y} \le 1$

$\frac{2y-1}{y}+1 \ge 0 \cap \frac{2y-1}{y}-1 \le 0$

$\frac{3y-1}{y} \ge 0 \cap \frac{y-1}{y} \le 0$

$\Rightarrow$ $y \ge \frac{1}{3} \cap y \le 1$

$\Rightarrow$ Range : $y \in \left[\frac{1}{3}, 1\right]$ $\leftarrow$

Alternate Method :

$$y=\frac{1}{2-\sin 3x}$$

we know, $-1 \le \sin 3x \le 1$

$\Rightarrow$ $1 \ge -\sin 3x \ge -1$

$\Rightarrow$ $1 \le 2-\sin 3x \le 3$

$\Rightarrow$ $\frac{1}{1} \ge \left(\frac{1}{2-\sin 3x}\right) \ge \frac{1}{3}$

$\Rightarrow$ Range $y \in \left[\frac{1}{3}, 1\right]$

Inequality changes upon reciprocating as all expressions across inequality are (positive).

Example – 38

Let $f, g : R \to R$ be defined respectively by $f(x)=x+1$, $g(x)=2x-3$. Find $f+g$, $f-g$ and f/g.

Sol. Let $f(x)=x+1, g(x)=2x-3$

$\therefore$ $f+g=f(x)+g(x)=(x+1)+(2x-3)$

$=3x-2$

$f-g=f(x)-g(x)=(x+1)-(2x-3)$

$=x+1-2x+3=-x+4$

$\frac{f}{g}=\frac{f(x)}{g(x)}=\frac{x+1}{2x-3};\ x \neq \frac{3}{2}$

Example – 39

Check whether the function :

$f(x) = 2x^3 + 3x^2 + 6x + 5$ is

one-to-one or many-to-one

Sol. $f(x) = 2x^3 + 3x^2 + 6x + 5$

$f'(x) = 6(x^2 + x + 1) > 0 \ \forall \ x \in R$

as $(a > 0 \ \& \ D < 0)$ for $x^2 + x + 1$

$\Rightarrow$ $f(x)$ is increasing function on its entire domain

$\Rightarrow$ one-to-one function.

Example – 40

Let $A = \{x : -1 \le x \le 1\} = B$ for a mapping $f : A \to B$. For the following functions from A to B, find whether it is surjective or bijective.

$f(x) = |x|$

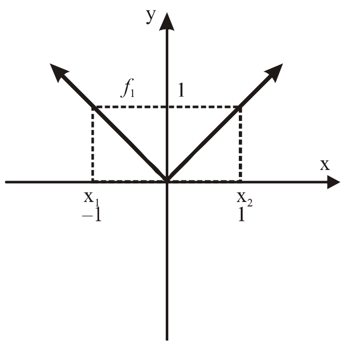
Sol. $f(x) = |x|$

Graphically we can see that for $x \in [-1, 1]$

$y = |x| \in [0, 1]$

Since, Range $([0, 1]) \subset$ co-domain $(B = [-1, 1])$

many-to-one



$\Rightarrow$ into function

$\Rightarrow$ $f : [-1, 1] \to [-1, 1], \ f(x) = |x|$

is many-to-one & into

Example – 41

Solve $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$

Sol. Here, $(x + 1)^2 + (x^2 + 3x + 2)^2 = 0$ if and only if each term is zero simultaneously,

$(x + 1) = 0$ and $(x^2 + 3x + 2) = 0$

i.e., $x = -1$ and $x = -1, -2$

The common solution is $x = -1$

Hence, solution of above equation is $x = -1$

Example – 42

Solve $\dfrac{|x+3| + x}{x+2} > 1$

Sol. $\dfrac{|x+3| + x}{x+2} - 1 > 0$

$\Rightarrow \dfrac{|x+3| + x - x - 2}{x+2} > 0$

$\Rightarrow \dfrac{|x+3| - 2}{x+2} > 0$ ...(i)

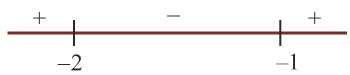
Now two cases arises :

Case I : When $x + 3 \ge 0$ ...(ii)

$\Rightarrow \dfrac{x + 3 - 2}{x + 2} > 0$

$\Rightarrow \dfrac{x + 1}{x + 2} > 0$

$\Rightarrow x \in (-\infty, -2) \cup (-1, \infty)$ using number line rule as shown in figure.



But $x \ge -3$ {from (ii)}

$\Rightarrow x \in [-3, -2) \cup (-1, \infty)$ ...(a)

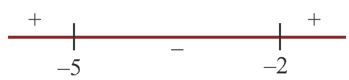
Case II : When $x + 3 < 0$ ...(iii)

$\Rightarrow \dfrac{-(x + 3) - 2}{x + 2} > 0$

$\Rightarrow \dfrac{-(x + 5)}{(x + 2)} > 0$

$\Rightarrow \dfrac{(x + 5)}{(x + 2)} < 0$

$\Rightarrow x \in (-5, -2)$ using number line rule as shown in figure.



But $\quad x < -3 \qquad$ {from (iii)}

$\therefore \quad x \in (-5, -3) \qquad$ ...(b)

Thus from (a) and (b), we have;

$x \in [-3, -2) \cup (-1, \infty) \cup (-5, -3)$

$\Rightarrow \quad x \in (-5, -2) \cup (-1, \infty)$

Example – 43

The value of x if $|x + 3| > |2x - 1|$ is

(a) $\left(-\frac{2}{3}, 4\right)$ (b) $\left(-\frac{2}{3}, \infty\right)$

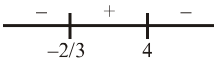
(c) (0, 1) (d) None of these

Sol. Squaring both sides, we get

$|x+3|^2 > |2x-1|^2$

or $\quad \{(x+3)-(2x-1)\}\ \{(x+3)+(2x-1)\} > 0$

$\Rightarrow \quad \{(-x+4)(3x+2)\} > 0$



$\Rightarrow \quad x \in \left(-\frac{2}{3}, 4\right)$

Hence, (a) is the correct answer.

Example – 44

Solve for x

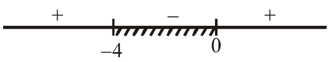
$$|x| + |x+4| = 4$$

Sol. $|x| + |x+4| = 4$

As we know, $\qquad |x| + |y| = |x - y|$, iff $xy \le 0$

$x(x+4) \le 0$

Using number line rule,



$\Rightarrow \quad x \in [-4, 0]$

Example – 45

Solve $\left|\frac{x}{x-1}\right| + |x| = \frac{x^2}{|x-1|}$

Sol. Let $f(x) = \frac{x}{x-1}$ and $g(x) = x$

$\therefore \quad f(x) + g(x) = \frac{x}{x-1} + x = \frac{x^2}{x-1}$

Using, $|f(x)| + |g(x)| = |f(x) + g(x)|$

i.e. $\qquad f(x) \cdot g(x) \ge 0$

$\Rightarrow \quad \frac{x}{x-1} \cdot x \ge 0 \ \Rightarrow \ \frac{x^2}{x-1} \ge 0$

$\Rightarrow \quad x \in \{0\} \cup (1, \infty)$

Example – 46

If y =3[x] +1 =2[x – 3] +5, then find the value of [x + y], where [.] represents greatest integer function.

Sol. We are given that $3[x] + 1 = 2([x] - 3) + 5$

$\Rightarrow \quad [x] = -2$

$\Rightarrow \quad y = 3(-2) + 1 = -5$

Hence $[x + y] = [x] + y = -2 - 5 = -7$

Example – 47

Solve the equation $|2x - 1| = 3[x] + 2\{x\}$ for x.

where [.] represents greatest integer function and {} represents fraction part function.

Sol. **Case I :** For $x < \frac{1}{2}$, $|2x - 1| = 1 - 2x$

$\Rightarrow \quad 1 - 2x = 3[x] + 2\{x\}$.

$\Rightarrow \quad 1 - 2x = 3(x - \{x\}) + 2\{x\}$.

$\Rightarrow \quad \{x\} = 5x - 1$.

Now $\ 0 \le \{x\} < 1$

$\Rightarrow \quad 0 \le 5x - 1 < 1$.

$\Rightarrow \quad \frac{1}{5} \le x < \frac{2}{5} \ \Rightarrow \ [x] = 0$

$\Rightarrow \quad x = \{x\} \Rightarrow \quad x = 5x - 1$

$\Rightarrow \quad x = \frac{1}{4}$, which is a solution.

Case II : For $x \ge \frac{1}{2}$, $|2x - 1| = 2x - 1$

$\Rightarrow \quad 2x - 1 = 3[x] + 2\{x\}$.

$\Rightarrow \quad 2x - 1 = 3(x - \{x\}) + 2\{x\}$.

$\{x\} = x + 1$

Now $0 \le \{x\} < 1$

$\Rightarrow \quad 0 \le x + 1 < 1. \Rightarrow \quad -1 \le x < 0.$

which is not possible since $x \ge \frac{1}{2}$.

Hence $x = \frac{1}{4}$ is the only solution.

Example – 48

For a real number x, $[x]$ denotes the integral part of x. The value of

$$\left[\frac{1}{2}\right]+\left[\frac{1}{2}+\frac{1}{100}\right]+\left[\frac{1}{2}+\frac{2}{100}\right]+....+\left[\frac{1}{2}+\frac{99}{100}\right] \text{is}$$

(a) 49 (b) 50

(c) 48 (d) 51

Ans. (b)

Sol. $\underbrace{\left[\frac{1}{2}+\frac{1}{100}\right]+........\left[\frac{1}{2}+\frac{49}{100}\right]}_{\textit{each has value}=0}+$

$\underbrace{\left[\frac{1}{2}+\frac{50}{100}\right]+........\left[\frac{1}{2}+\frac{99}{100}\right]}_{\textit{each has value}=1}$

$= 50$

Example – 49

Find the domain of definition of the following

function : $f(x) = \sqrt{\log_{\frac{1}{2}}(2x-3)}$

Sol. For f(x) to be defined $\log_{1/2}(2x - 3) \ge 0$

$\Rightarrow \quad 2x - 3 \le 1$

$\Rightarrow \quad x \le 2 \quad ....(1)$

Also $2x - 3 > 0$

$\Rightarrow \quad x > \frac{3}{2}. \quad ....(2)$

Combining (1) and (2) we get the required values of x.

Hence the domain of definition of f(x) is the set $\left(\frac{3}{2}, 2\right]$

Example – 50

Find the domain of the function;

$$f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{(x+2)}$$

Sol. $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

{as we know; $\log_a x$ is defined when x and a > 0 and a ≠ 1 also $\log_a 1 = 0$}

Thus, $\log_{10}(1 - x)$ exists when, $1 - x > 0 \quad$ ...(i)

also $\frac{1}{\log_{10}(1-x)}$ exists when, $1 - x > 0$

and $\quad 1 - x \ne 1 \quad$ ...(ii)

$\Rightarrow \quad x < 1$ and $x \ne 0 \quad$ ...(iii)

also we have $\sqrt{x+2}$ exists when $x + 2 \ge 0$

or $\quad x \ge -2 \quad$ ....(iv)

Thus, $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$ exists when (iii) and (iv) both holds true.

$\Rightarrow \quad -2 \le x < 1$ and $x \ne 0$

$\Rightarrow \quad x \in [-2, 0) \cup (0, 1)$

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Types of sets & Subset

1. The set of intelligent students in a class is

(a) a null set

(b) a singleton set

(c) a finite set

(d) not a well defined collection

2. Which of the following is the empty set?

(a) $\{x : x \text{ is a real number and } x^2 - 1 = 0\}$

b) $\{x : x \text{ is a real number and } x^2 + 1 = 0\}$

(c) $\{x : x \text{ is a real number and } x^2 - 9 = 0\}$

(d) $\{x : x \text{ is a real number and } x^2 = x + 2\}$

3. Let $A = \{2, 3, 4\}$ and $X = \{0, 1, 2, 3, 4\}$, then which of the following statements is correct

(a) $\{0\} \in A^c$ w.r.t. X (b) $\phi \in A^c$ w.r.t. X

(c) $\{0\} \subset A^c$ w.r.t.X (d) $0 \subset A^c$ w.r.t. X.

Operation on sets

4. If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, \ldots\ldots, 18\}$ and N is the universal set, then $A^c \cup ((A \cup B) \cap B^c)$ is

(a) A (b) N

(c) B (d) None of these

5. Let $A = \{x : x \text{ is a multiple of } 3\}$ and $B = \{x : x \text{ is a multiple of } 5\}$. Then $A \cap B$ is given by

(a) $\{3, 6, 9\ldots\}$ (b) $\{5, 10, 15, 20, \ldots\}$

(c) $\{15, 30, 45, \ldots\}$ (d) None of these

6. If $Y \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$, then

(a) The smallest set of Y is $\{3, 5, 9\}$

(b) The smallest set of Y is $\{2, 3, 5, 9\}$

(c) The largest set of Y is $\{1, 2, 3, 5\}$

(d) The largest set of Y is $\{2, 3, 5, 9\}$

7. Given the sets $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $[A \cup (B \cap C)]$ is

(a) $\{1, 2, 3, 4, 5, 6\}$ (b) $\{1, 2, 4, 5\}$

(c) $\{1, 2, 3, 4\}$ (d) $\{3\}$

8. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to

(a) B^c (b) A^c

(c) B (d) A

9. If $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 5, 6\}$ and $C = \{3, 4, 6, 7\}$, then

(a) $A - (B \cap C) = \{1, 3, 4\}$ (b) $A - (B \cap C) = \{1, 2, 4\}$

(c) $A - (B \cup C) = \{2, 3\}$ (d) $A - (B \cup C) = \{1, 2\}$

Classification of function

10. Let $A = [-1, 1]$ and $f : A \to A$ be defined as $f(x) = x|x|$ for all $x \in A$, then $f(x)$ is

(a) many-one into function

(b) one-one into function

(c) many-one onto function

(d) one-one onto function

11. The function $f : R \to R$ defined by

$f(x) = (x-1)(x-2)(x-3)$ is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one nor onto

12. Let $f : R \to R$ be defined by $f(x) = \dfrac{|x|-1}{|x|+1}$ then f is:

(a) both one - one and onto

(b) one - one but not onto

(c) onto but not one - one

(d) neither one - one nor onto.

13. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when n is odd} \\ -\dfrac{n}{2}, & \text{when n is even} \end{cases} \quad \text{is}$$

(a) onto but not one-one

(b) one-one and onto both

(c) neither one-one nor onto

(d) one-one but not onto

Domain of a function

14. Find the domain of $f(x)=\frac{1}{\sqrt{x-5}}+x^2+\frac{1}{\sqrt{x+7}}$

(a) $x \in [-7, 5]$ (b) $x \in (5, \infty)$
(c) $x \in (-\infty, 7)$ (d) none of these

15. Find the domain $y=\sqrt{1-x}+\sqrt{x-5}$

(a) $x \in \phi$ (b) $y \in (-\infty, 1]$
(c) $x \in (-\infty, 1] \cup [5, \infty)$ (d) none of these

16. The domain of the function

$f(x)=\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$ is

(a) $[4, \infty)$ (b) $(-\infty, 4]$
(c) $(4, \infty)$ (d) $(-\infty, 4)$

17. If $f(x)=\frac{1}{\sqrt{|x|-x}}$, then domain of $f(x)$ is

(a) $(-\infty, 0)$ (b) $(-\infty, 2)$
(c) $(-\infty, \infty)$ (d) None of the above

Modulus functions

18. $|3x+7| < 5$, then x belongs to

(a) $(-4, -3)$ (b) $(-4, -2/3)$
(c) $(-5, 5)$ (d) $(-5/3, 5/3)$

19. Solution of $|3x-2| \geq 1$ is

(a) $[1/3, 1]$ (b) $(1/3, 1)$
(c) $\{1/3, 1\}$ (d) $\left(-\infty, \frac{1}{3}\right] \cup [1, \infty)$

20. If $-5 < x < 4$, then :

(a) $0 \leq |x| < 4$ (b) $4 < |x| < 5$
(c) $0 \leq |x| < 5$ (d) none of these

21. $|2x-3| < |x+5|$, then x belongs to

(a) $(-3, 5)$ (b) $(5, 9)$
(c) $(-2/3, 8)$ (d) $(-8, 2/3)$

22. $\left|\frac{x^2+6}{5x}\right| \geq 1$

(a) $(-\infty, -3)$

(b) $(-\infty, -3) \cup (3, \infty)$

(c) R

(d) $(-\infty, -3] \cup [-2, 0) \cup (0, 2] \cup [3, \infty)$

23. Solution of $\left|x+\frac{1}{x}\right| < 4$ is

(a) $\left(2-\sqrt{3}, 2+\sqrt{3}\right) \cup \left(-2-\sqrt{3}, -2+\sqrt{3}\right)$

(b) $R-\left(2-\sqrt{3}, 2+\sqrt{3}\right)$

(c) $R-\left(-2-\sqrt{3}, -2+\sqrt{3}\right)$

(d) none of these

Greatest integer functions

24. The domain of the function

$$f(x)=\frac{1}{\sqrt{[x]^2-[x]-6}}$$

where [ ] denotes greatest integer function

(a) $R-[-2, 4)$ (b) $R-\{-3, 2\}$
(c) R (d) $R-\{2, 3\}$

25. If $[x]^2=[x+2]$, where $[x]$ = the greatest integer less than or equal to x, then x must be such that

(a) $x=2, -1$ (b) $x \in [2, 3)$
(c) $x \in [-1, 0)$ (d) none of these

26. The domain of the function $f(x)=\log_e(x-[x])$, where [.] denotes the greatest integer function, is

(a) R (b) R–Z
(c) $(0, +\infty)$ (d) None of these

Logarithmic functions

27. Let $f(x)=\log_{x^2} 25$ and $g(x)=\log_x 5$ then $f(x)=g(x)$ holds for x belonging to

(a) R (b) $(0, 1) \cup (1, +\infty)$
(c) ϕ (d) None of these

28. The domain of the function $f(x)=\log_2(\log_3(\log_4 x))$ is

(a) $(-\infty, 4)$ (b) $(4, \infty)$
(c) $(0, 4)$ (d) $(1, \infty)$

29. The value of x, $\log_{1/2} x \geq \log_{1/3} x$ is

(a) $(0, 1]$ (b) $(0, 1)$
(c) $[0, 1)$ (d) none

30. Indicate the correct alternative : The number $\log_2 7$ is

(a) an integer (b) a rational number
(c) an irrational number (d) a prime number

Range of a function

31. Find the Range $y = \frac{2x+1}{x-5}$

(a) $R - \{2\}$ (b) $x \neq 5$

(c) $R - \{5\}$ (d) none of these

32. Range of the function $f(x) = \frac{x}{1+x^2}$ is

(a) $(-\infty, \infty)$ (b) $[-1, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (d) $\left[-\sqrt{2}, \sqrt{2}\right]$

33. The range of the function $f(x) = x^2 + \frac{1}{x^2+1}$

(a) $[1, \infty)$ (b) $[2, \infty)$

(c) $\left[\frac{3}{2}, \infty\right)$ (d) None of these

Wavy Curve Method

34. The set of values of x satisfying the inequalities $(x-1)(x-2) < 0$ and $(3x-7)(2x-3) > 0$ is

(a) $(1, 2)$ (b) $(2, 7/3)$

(c) $(1, 7/3)$ (d) $(1, 3/2)$

35. If $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0$, then

(a) $x > 3$ (b) $x < 3$

(c) $3 < x < 4$ (d) $x = 7/2$

36. Find the set of values of 'x' for which the given condition is true $(x-1)(x-3)(x+5) > 0$

(a) $(-5, 1) \cup (3, \infty)$ (b) $(-1, 5)$

(c) $[-5, 1] \cup [3, \infty)$ (d) none of these

37. The value of x for which $12x - 6 < 0$ and $12 - 3x < 0$

(a) ϕ (b) R

(c) $R - \{0\}$ (d) none of these

38. The value of x for which $\frac{x-3}{4} - x < \frac{x-1}{2} - \frac{x-2}{3}$ and $2 - x > 2x - 8$

(a) $[-1, 10/3]$ (b) $(-1, 10/3)$

(c) R (d) none of these

39. If $c < d$, $x^2 + (c+d)x + cd < 0$, then x belongs to.

(a) $(-d, -c]$ (b) $(-d, -c)$

(c) R (d) ϕ

40. Solution of $\frac{x-7}{x+3} > 2$ is

(a) $(-3, \infty)$ (b) $(-\infty, -13)$

(c) $(-13, -3)$ (d) none of these

41. The set of values of x which satisfy the inequations $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is

(a) $(-\infty, 1)$ (b) $(2, 3)$

(c) $(-\infty, 3)$ (d) $(-\infty, 1) \cup (2, 3)$

42. If $x^2 - 1 \leq 0$ and $x^2 - x - 2 \geq 0$, then x lies in the interval set

(a) $(1, -1)$ (b) $(-1, 1)$

(c) $(1, 2)$ (d) $\{-1\}$

43. The solution set of $\frac{x^2 - 3x + 4}{x+1} > 1$, $x \in R$ is

(a) $(3, \infty)$ (b) $(-1, 1) \cup (3, \infty)$

(c) $[-1, 1] \cup [3, \infty)$ (d) none

44. If $\frac{1}{a} < \frac{1}{b}$, then :

(a) $|a| > |b|$ (b) $a < b$

(c) $a > b$ (d) none of these

45. If $-2 < x < 3$, then :

(a) $4 < x^2 < 9$ (b) $0 \leq |x| < 5$

(c) $0 \leq x^2 < 9$ (d) None of these

46. $x > \sqrt{2 - x^2}$

(a) $x \in (1, \infty)$ (b) $x \in (-\infty, -1) \cup (1, \infty)$

(c) $x \in \left(1, \sqrt{2}\right]$ (d) $x \in \left[\sqrt{2}, \infty\right)$

Misc examples-sets-functions

47. A function whose graph is symmetrical about the y-axis is given by

(a) $f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

(b) $f(x+y) = f(x) + f(y)$ for all $x, y \in R$

(c) $f(x) = \cos x + \sin x$

(d) None of these

48. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then

(a) $f(x) = f(-x)$ (b) $f(2+x) = f(2-x)$

(c) $f(x+2) = f(x-2)$ (d) $f(x) = -f(-x)$

49. If $A = \{1, 2, 3\}$, $B = \{a, b\}$, then $A \times B$ is given by

(a) $\{(1, a), (2, b), (3, b)\}$

(b) $\{(1, b), (2, a)\}$

(c) $\{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

(d) $\{(1, a), (2, a), (2, b), (3, b)\}$

50. Solve for x : $3^{(x^2-2)} < \left(\frac{1}{3}\right)^{\left(1-\frac{3}{2}|x|\right)}$

(a) $\left(-\sqrt{2}, -1\right)$ (b) $\left(-\sqrt{2}, 2\right)$

(c) $\left(-2, -\sqrt{2}\right)$ (d) None of these

51. The largest interval among the following for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is

(a) $-4 < x \le 0$ (b) $0 < x < 1$

(c) $-100 < x < 100$ (d) $-\infty < x < \infty$

52. If $f(x) = x^2 - 3x + 1$ and $f(2\alpha) = 2f(\alpha)$, then α is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}}$ (d) none of these

Numerical Value Type Questions

53. If $A = \{x : x = 4n + 1, 2 \le n \le 5\}$, $n \in N$ then number of subsets of A is

54. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \ne +1\}$. Then the number of elements in the power set of R is:

55. In a city 20% of the population travels by car, 50% travels by bus and 10% travels by both car and bus. Then, percentage of persons travelling by car or bus is

56. X and Y are two sets such that n (X) = 17, n(Y) = 23, $n(X \cup Y) = 38$ then $n(X \cap Y)$ is

57. If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, then number of elements $S \cup T$ has

58. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. The number of persons speaking at least one of these two languages is

59. In a group of 1000 people, there are 750 people who can speak Hindi and 400 who can speak English .Then number of persons who can speak Hindi only is

60. In a statistical investigation of 1,003 families of Calcutta, it was found that 63 families had neither a radio nor a T.V, 794 families had a radio and 187 had a T.V. The number of families in that group having both a radio and a T.V is

61. If A has 3 elements and B has 6 elements, then the minimum number of elements in the set $A \cup B$ is

62. If the value for which $\frac{(x-1)}{x} \ge 2$ is $[-k, 0)$, then the value of k is

63. If the domain of the function $f(x) = \sqrt{\log_{10}\frac{3-x}{x}}$ is $(a, b]$ then $a + 2b$ equals

64. The number of real solutions of

$$\sqrt{x^2-4x+3} = \sqrt{x^2-9} = \sqrt{4x^2-14x+6} \text{ is}$$

65. The number of real solutions of the equation $e^x = x$ is

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. Let $S = \{x \in R : x \geq 0$ and $2\left|\sqrt{x}-3\right|+\sqrt{x}\left(\sqrt{x}-6\right)+6=0\}$ Then S : **(2018)**

(a) Contain exactly four element

(b) is an empty set.

(c) contain exactly one element

(d) contains exactly two elements.

2. Let $f(x) = a^x\ (a > 0)$ be written as $f(x) = f_1(x) + f_2(x)$, when $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y)+f_2(x-y)$ equals : **(8-04-2019/Shift-2)**

(a) $2f_1(x)f_1(y)$ (b) $2f_1(x+y)f_1(x-y)$

(c) $2f_1(x)f_2(y)$ (d) $2f_1(x+y)f_2(x-y)$

3. The domain of the definition of the function

$$f(x)=\frac{1}{4-x^2}+\log_{10}\left(x^3-x\right) \text{ is:}$$

(9-04-2019/Shift-2)

(a) $(-1,0)\cup(1,2)\cup(3,\infty)$

(b) $(-2,-1)\cup(-1,0)\cup(2,\infty)$

(c) $(-1,0)\cup(1,2)\cup(2,\infty)$

(d) $(1,2)\cup(2,\infty)$

4. Two newspapers A and B are published in city. It is known that 25% of the city population reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisements is: **(9-04-2019/Shift-2)**

(a) 13.9 (b) 12.8

(c) 13 (d) 13.5

5. Let A, B and C be sets such that $\phi \neq A\cap B \subseteq C$. Then which of the following statements is not true ? **(12-04-2019/Shift-2)**

(a) $B\cap C\neq\phi$

(b) If $(A-B\subseteq C)$, then $A\subseteq C$

(c) $(C\cup A)\cap(C\cup B)=C$

(d) If $(A-C)\subseteq B$, then $A\subseteq B$

6. Let A = $\{x \in R$: x is not a positive integer$\}$. Define a function f: $A \to R$ as $f(x)=\frac{2x}{x-1}$, then f is: **(9-01-2019/Shift-2)**

(a) not injective

(b) neither injective nor surjective

(c) surjective but not injective

(d) injective but not surjective

7. In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is: **(10-01-2019/Shift-1)**

8. Let N be the set of natural numbers and two functions f and g be defined as $f, g : \mathbf{N} \to \mathbf{N}$ such that

$$f(n)=\begin{cases}\frac{n+1}{2} & \text{if } n \text{ is odd}\\ \frac{n}{2} & \text{if } n \text{ is even}\end{cases}$$

and $g(n)=n-(-1)^n$. Then fog is: **(10-01-2019/Shift-2)**

(a) onto but not one-one

(b) one-one but not onto

(c) both one-one and onto

(d) neither one-one nor onto

9. Let $f: R \to R$ be defined by $f(x)=\frac{x}{1+x^2}, x \in R$.

Then the range of f is : **(11-01-2019/Shift-1)**

(a) $\left[-\frac{1}{2},\frac{1}{2}\right]$ (b) $R-[-1,1]$

(c) $R-\left[-\frac{1}{2},\frac{1}{2}\right]$ (d) $(-1,1)-\{0\}$

10. Let $S=\{1,2,3,\ldots,100\}$. The number of non-empty subsets A of S such that the product of elements in A is even is : **(12-01-2019/Shift-1)**

(a) $2^{100}-1$ (b) $2^{50}\left(2^{50}-1\right)$

(c) $2^{50}-1$ (d) $2^{50}+1$

11. Let Z be the set of integers.

If $A=\left\{x \in Z : 2^{(x+2)\left(x^2-5x+6\right)}=1\right\}$ and

$B=\left\{x \in Z : -3<2x-1<9\right\}$ then the number of subsets of the set A x B, is **(12-01-2019/Shift-2)**

(a) 2^{15} (b) 2^{18}

(c) 2^{12} (d) 2^{10}

12. If $R=\{(x,y): x,y \in Z, x^2+3y^2 \le 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is : **(2-9-2020/Shift-1)**

(a) $\{-1,0,1\}$ (b) $\{-2,-1,1,2\}$

(c) $\{0,1\}$ (d) $\{-2,-1,0,1,2\}$

13. Let [t] denote the greatest integer $\le$ t. Then the equation in x, $[x]^2+2[x+2]-7=0$ has : **(4-09-2020/Shift-1)**

(a) exactly four integral solutions

(b) infinitely many solutions

(c) no integral solution

(d) exactly two solution

14. A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x % of the people read both the newspapers, then a possible value of x can be : **(4-09-2020/Shift-1)**

(a) 37 (b) 29

(c) 65 (d) 55

15. A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be: **(5-09-2020/Shift-1)**

(a) 63 (b) 54

(c) 38 (d) 36

16. Set A has m elements and set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of m.n is _____.

(6-09-2020/Shift-1)

17. Let $X=\{n \in N : 1 \le n \le 50\}$. If A= {n ∈ X: n is a multiple of 2} and B = {n∈X: n is a multiple of 7}, then the number of elements in the smallest subset of X containing both A and B is _______. **(7-01-2020/Shift-2)**

18. Let f: $(1,3) \to$ R be a function defined by $f(x)=\frac{x[x]}{x^2+1}$,

where $[x]$ denotes the greatest integerd $\le x$ Then the range of f is: **(8-01-2020/Shift-2)**

(a) $\left(\frac{2}{5},\frac{3}{5}\right]\cup\left(\frac{3}{4},\frac{4}{5}\right)$ (b) $\left(\frac{2}{5},\frac{4}{5}\right]$

(c) $\left(\frac{3}{5},\frac{4}{5}\right)$ (d) $\left(\frac{2}{5},\frac{1}{2}\right)\cup\left(\frac{3}{5},\frac{4}{5}\right]$

19. If $A=\{x \in R : |x|<2\}$ and $B=\{x \in R : |x-2| \ge 3\}$ then :

(9-1-2020/Shift-2)

(a) $A-B=[-1,2]$ (b) $B-A=R-(-2,5)$

(c) $A \cup B = R-(2,5)$ (d) $A \cap B=(-2,-1)$

20. Let $A = \{n \in N : n \text{ is a } 3-\text{digit number}\}$

$B = \{9k+2 : k \in N\}$ and

$C = \{9k+l : k \in N\}$ for some $l(0<l<9)$

If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ______ **(24-02-2021/Shift-1)**

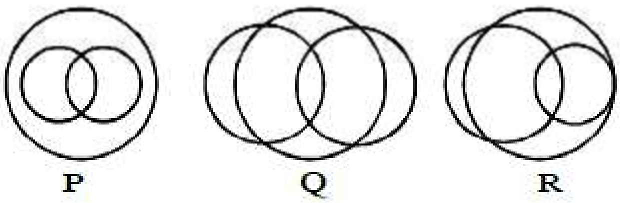
21. The number of elements in the set $\{x \in R : (|x|-3)|x+4| = 6\}$ is equal to

(16-03-2021/Shift-1)

(a) 1 (b) 3

(c) 2 (d) 4

22. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement ?



(17-03-2021/Shift-1)

(a) P and R (b) None of these

(c) Q and R (d) P and Q

23. Let $A = \{n \in N \mid n^2 \leq n + 10{,}000\}, B = \{3k+1 \mid k \in N\}$ and $C = \{2k \mid k \in N\}$, then the sum of all the elements of the set $A \cap (B - C)$ is equal to ______.

(27-07-2021/Shift-2)

24. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If $K\%$ of them are suffering from both ailments, then K cannot belong to the set:

(26-08-2021/Shift-1)

(a) $\{80, 83, 86, 89\}$ (b) $\{79, 81, 83, 85\}$

(c) $\{84, 87, 90, 93\}$ (d) $\{84, 86, 88, 90\}$

25. If $A = \{x \in R : |x-2| > 1\}$, $B = \left\{x \in R : \sqrt{x^2-3} > 1\right\}$, $C = \{x \in R : |x-4| \geq 2\}$ and Z is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap Z$ is _______. **(27-08-2021/Shift-1)**

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Objective Questions I [Only one correct option]

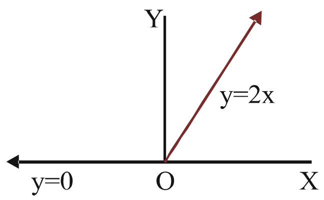
1. The solution set of $\log_2 |4-5x| > 2$ is

(a) $\left(8/5, \infty\right)$ (b) $\left(4/5, 8/5\right)$

(c) $(-\infty, 0)\cup\left(8/5, \infty\right)$ (d) none

2. The graph of a real-valued function $f(x)$ is the following.

The function is



(a) $f(x)=x-|x|$ (b) $f(x)=x+|x|$

(c) $f(x)=2x$ (d) None of these

3. Solution of the inequality $x > \sqrt{(1-x)}$ is given by

(a) $\left(-\infty, \left(-1-\sqrt{5}\right)/2\right)$

(b) $\left(\left(\sqrt{5}-1\right)/2, \infty\right)$

(c) $\left(-\infty, \left(-1-\sqrt{5}\right)/2\right)\cup\left(\left(\sqrt{5}-1\right)/2, \infty\right)$

(d) $\left(\left(\sqrt{5}-1\right)/2, 1\right]$

4. If for $x \in R$, $\frac{1}{3} \le \frac{x^2-2x+4}{x^2+2x+4} \le 3$, then $\frac{9.3^{2x}+6.3^{x}+4}{9.3^{2x}-6.3^{x}+4}$ lies b/w

(a) 1 and 2 (b) 1/3 and 3

(c) 0 and 4 (d) none of these

5. The domain of the function $f(x)=\sqrt{x-\sqrt{1-x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}}, 1\right]$

(b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right]\cup\left[\frac{1}{\sqrt{2}}, \infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

6. The function $f: R \to R$ defined by $f(x) = 6^x + 6^{|x|}$ is

(a) one-one and onto (b) many-one and onto

(c) one-one and into (d) many-one and into

7. If $|x-1|+|x|+|x+1| \ge 6$; then x lies in

(a) $(-\infty, 2]$ (b) $(-\infty, -2]\cup[2, \infty)$

(c) R (d) ϕ

8. Solution of $|1/x-2|<4$ is

(a) $\left(-\infty, -1/2\right)$ (b) $\left(1/6, \infty\right)$

(c) $\left(-1/2, 1/6\right)$ (d) $\left(-\infty, -1/2\right)\cup\left(1/6, \infty\right)$

9. Solution of $2^x + 2^{|x|} \ge 2\sqrt{2}$ is

(a) $\left(-\infty, \log_2\left(\sqrt{2}+1\right)\right)$

(b) $(0, \infty)$

(c) $\left(\frac{1}{2}, \log_2\left(\sqrt{2}-1\right)\right)$

(d) $\left(-\infty, \log_2\left(\sqrt{2}-1\right)\right]\cup\left[1/2, \infty\right)$

10. If $f(x) = \cos[\pi]x + \cos[\pi x]$, where [y] is the greatest integer function of y then $f(\pi/2)$ is equal to

(a) cos 3 (b) 0

(c) cos 4 (d) none of these

11. The domain of the function $f(x) = \sqrt{x^2 - [x]^2}$, where $[x]$ = the greatest integer less than or equal to x is

(a) R (b) $[0, +\infty)$

(c) $(-\infty, 0]$ (d) None of these

12. Let $f(x) = [x]$ = the greatest integer less than or equal to x and $g(x) = x - [x]$. Then for any two real numbers x and y.

(a) $f(x+y) = f(x) + f(y)$

(b) $g(x+y) = g(x) + g(y)$

(c) $f(x+y) = f(x) + f(y + g(x))$

(d) none of these

13. The domain of $f(x) = \sqrt{\log_{x^2-1}(x)}$ is

(a) $(\sqrt{2}, +\infty)$ (b) $(0, \infty)$

(c) $(1, +\infty)$ (d) None of these

14. The domain of the real-valued function $f(x) = \log_e |\log_e x|$ is

(a) $(1, +\infty)$ (b) $(0, +\infty)$

(c) $(e, +\infty)$ (d) None of these

15. If $x = \log_a(bc)$, $y = \log_b(ca)$ and $z = \log_c(ab)$ then which of the following is equal to 1?

(a) $x + y + z$ (b) $(1+x)^{-1} + (1+y)^{-1} + (1+z)^{-1}$

(c) xyz (d) none of these

16. If $f(n+1) = \dfrac{2f(n)+1}{2}$, n = 1, 2, ... and $f(1) = 2$, then $f(101)$ equals

(a) 52 (b) 49

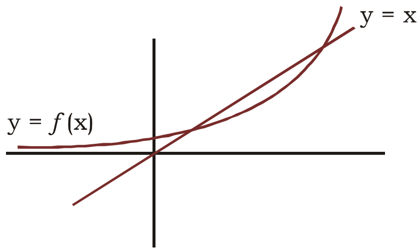
(c) 48 (d) 51

17. The domain of function $f(x) = \dfrac{1}{\sqrt{x^2 - \{x\}^2}}$, where $\{x\}$ denotes fraction part of x.

(a) $R - [0, 1)$ (b) $R - \left[\frac{1}{2}, 1\right]$

(c) $(-\infty, \frac{1}{2}] \cup (1, \infty)$ (d) none of these

18. If graph of $y = f(x)$ is



Then $f(x)$ can be

(a) $y = 2e^x$ (b) $y = 4e^x$

(c) $y = e^{x+\frac{1}{2}}$ (d) $y = \frac{1}{4}e^x$

19. The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$ is

(a) $0 < x \le 1$ (b) $0 \le x \le 1$

(c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

20. Solution set of the inequality : $\dfrac{1}{2^x - 1} > \dfrac{1}{1 - 2^{(x-1)}}$ is

(a) $(1, \infty)$ (b) $\left(0, \log_2\left(4/3\right)\right)$

(c) $(-1, \infty)$ (d) $\left(0, \log_2\left(4/3\right)\right) \cup (1, \infty)$

21. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in the interval

(a) $\left[-\frac{1}{2}, 1\right]$ (b) $\left[0, \frac{1}{2}\right]$

(c) $[0, 1]$ (d) $[1, 2]$

22. Let f: {x, y, z} $\to$ {a, b, c} be a one-one function and only one of the conditions (i) $f(x) \ne b$, (ii) $f(y)$, = b (iii) $f(z) \ne a$ is true then the function f is given by the set

(a) {(x, a), (y, b), (z, c)} (b) {(x, a), (y, c), (z, b)}

(c) {(x, b), (y, a), (z, c)} (d) {(x, c), (y, b), (z, a)}

23. The equation $||x-1| + a| = 4$ can have real solutions for x if **'a'** belongs to the interval

(a) $(-\infty, 4]$ (b) $(-\infty, -4]$

(c) $(4, +\infty)$ (d) $[-4, 4]$

24. If $x^4 f(x) - \sqrt{1 - \sin 2\pi x} = |f(x)| - 2f(x)$, then f(-2) equals:

(a) $\frac{1}{17}$ (b) $\frac{1}{11}$

(c) $\frac{1}{19}$ (d) 0

25. If $0 < x < 1000$ and $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{5}\right]=\frac{31}{30}x$, where $[x]$ is the greatest integer less than or equal to x, the number of possible values of x is

(a) 34 (b) 32

(c) 33 (d) none of these

26. The domain of the function $y = \underbrace{\log_{10}\log_{10}\log_{10}\ldots\log_{10}}_{n \text{ times}} x$ is

(a) $[10^n, +\infty)$ (b) $(10^{n-1}, +\infty)$

(c) $[10^{n-2}, +\infty)$ (d) None of these

27. If [.] denotes the greatest integer function then the domain of the real-valued function $\log_{[x+1/2]}|x^2 - x - 2|$ is

(a) $\left[\frac{3}{2}, +\infty\right)$ (b) $\left[\frac{3}{2}, 2\right) \cup (2, +\infty)$

(c) $\left(\frac{1}{2}, 2\right) \cup (2, +\infty)$ (d) None of these

Objective Questions II [One or more than one correct option]

28. If $\log_k x \cdot \log_5 k = \log_x 5$, $k \neq 1$, $k > 0$, then x is equal to

(a) k (b) $\frac{1}{5}$

(c) 5 (d) None of these

29. If $\frac{1}{2} \leq \log_{0.1} x \leq 2$ then

(a) the maximum value of x is $\frac{1}{\sqrt{10}}$

(b) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(c) x does not lie between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

(d) the minimum value of x is $\frac{1}{100}$

30. Which of the following functions is not injective ?

(a) $f(x) = |x+1|, x \in [-1, 0]$

(b) $f(x) = x + 1/x,\ x \in (0, \infty)$

(c) $f(x) = x^2 + 4x - 5$

(d) $f(x) = e^{-x},\ x \in [0, \infty)$

31. If f is an even function defined on the interval $(-5, 5)$ then a value of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ is

(a) $\frac{-1+\sqrt{5}}{2}$ (b) $\frac{-3+\sqrt{5}}{2}$

(c) $\frac{-1-\sqrt{5}}{2}$ (d) $\frac{-3-\sqrt{5}}{2}$

Numerical Value Type Questions

32. If $f\left(x+\frac{1}{x}\right) = x^3 + x^{-3}$ then f(5) must be equal to

33. The range of the function $\sqrt{x-6} + \sqrt{12-x}$ is an interval of length $2\sqrt{3} - \sqrt{k}$ then k must be

34. The least period of the function

$\cos(\cos x) + \sin(\cos x) + \sin 4x$ is $k\frac{\pi}{2}$

then value of k must be

Assertion & Reason

(A) **If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.**

(B) **If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.**

(C) **If ASSERTION is true, REASON is false.**

(D) **If ASSERTION is false, REASON is true.**

35. **Assertion :** The function $\frac{ax+b}{cx+d}$, $(ad - bc \neq 0)$ cannot attain the value $\frac{a}{c}$.

Reason : The domain of the function $g(y) = \frac{b-dy}{cy-a}$ is all the reals except a/c.

(a) A (b) B

(c) C (d) D

36. **Assertion :** The domain of a function $y = f(x)$ will be all reals if for every real x there exist real y.

Reason : The range of a function $y = f(x)$ will be all reals if for every real y there exists a real x such that $f(x) = y$.

(a) A (b) B

(c) C (d) D

37. **Assertion :** Let A and B be two sets each with a finite number of elements. Assume that there is an injective mapping from A to B and that there is an injective mapping from B to A. Then there is a bijective mapping from A to B.

Reason : An onto function is not necessarily one-one.

(a) A (b) B

(c) C (d) D

Match the Following

Each question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching.For each question, choose the option corresponding to the correct matching.

38.

Column–I	**Column–II**
(A) odd function	**(P)** $x-[x]$
(B) even function	**(Q)** $\log\left(x+\sqrt{1+x^2}\right)$
(C) neither even nor odd	**(R)** $x\log\dfrac{1+x}{1-x}$
	(S) $\dfrac{2^{x/2}}{1+2^{x/2}}$

The Correct Matching is :

(a) (A–R); (B–Q); (C–P,S)

(b) (A–P,S); (B–R); (C–R)

(c) (A–Q); (B–P,S); (C–R)

(d) (A–Q); (B–R); (C–P,S)

39.

Column–I	**Column–II**
(A) $f(x+y)=f(x)+f(y)$	**(P)** $\log_3 x$
(B) $f(xy)=f(x)+f(y)$	**(Q)** $\tan^{-1}x$
(C) $f(x+y)=f(x).f(y)$	**(R)** $3x$
(D) $f(x)+f(y)=f\left(\dfrac{x+y}{1-xy}\right)$	**(S)** 3^x

The Correct Matching is :

(a) (A–P); (B–R); (C–S); (D–Q)

(b) (A–R); (B–P); (C–S); (D–Q)

(c) (A–Q); (B–S); (C–P); (D–R)

(d) (A–R); (B–S); (C–Q); (D–P)

Using the following passage, solve Q.40 to Q.42

Passage

A rational function is defined as quotient of two polynomials, p(x) and q(x). The domain of the rational function must be all reals except the roots of the equation q(x) = 0. The range of rational function can be found by finding minimum and maximum values of the function. In case p(x) and q(x) have a common factor $x-\beta$. Then after cancelling the common factor, the rational function must assume a value at $x = \beta$ which should be deleted from the found range since β is not there in the domain of the rational function.

40. The range of the rational function $f(x) = \dfrac{3x+1}{2x+1}$ must be

(a) $R - \left\{-\dfrac{1}{2}\right\}$ (b) $R - \left\{-\dfrac{1}{3}\right\}$

(c) $R - \left\{\dfrac{3}{2}\right\}$ (d) R

41. The range of the rational function $f(x) = \dfrac{(2x+1)}{2x^2+5x+2}$ must be

(a) $R - \{0\}$ (b) $R - \{-2\}$

(c) $R - \{0, -2\}$ (d) $R - \left\{0, \dfrac{2}{3}\right\}$

42. The range of the rational function $f(x) = \dfrac{2x^2 + 5x + 2}{2x + 1}$ must be

(a) $R - \{0\}$ (b) $R - \{-2\}$

(c) $R - \left\{0, -2, \dfrac{2}{3}\right\}$ (d) $R - \left\{\dfrac{3}{2}\right\}$

Text

43. Find all real numbers x which satisfy the equation,

$2\, log_2\, log_2\, x + log_{1/2}\, log_2\left(2\sqrt{2}\, x\right) = 1.$

44. Find the values of x satisfying the equation

$|x-1|^A = (x-1)^7$ where $A = log_3 x^2 - 2\, log_x 9.$

45. Find all real numbers x which satisfy the equation,

$\log_{3/4} \log_8 (x^2+7) + \log_{1/2} \log_{1/4} (x^2+7)^{-1} = -2.$

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Objective Questions I [Only one correct option]

1. Number of solutions of $\log_4(x-1) = \log_2(x-3)$ is **(2001)**

(a) 3 (b) 1
(c) 2 (d) 0

2. If $f:[0,\infty) \to [0,\infty)$ and $f(x) = \frac{x}{1+x}$, then f is **(2003)**

(a) one-one and onto
(b) one-one but not onto
(c) onto but not one-one
(d) neither one-one nor onto

3. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}; x \in R$ is **(2003)**

(a) $(1, \infty)$ (b) $(1, 11/7)$
(c) $(1, 7/3]$ (d) $(1, 7/5)$

4. The function $f:[0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **(2012)**

(a) one-one and onto
(b) onto but not one-one
(c) one-one but not onto
(d) neither one-one nor onto

Match the Following

The question has two columns. Four options are given representing matching of elements from Column-I and Column-II. Only one of these four options corresponds to a correct matching. For question, choose the option corresponding to the correct matching.

Match the conditions/expressions in Column I with statement in Column II.

5. Let $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$

	Column I	Column II
(A)	If $-1 < x < 1$, then $f(x)$ satisfies	**(P)** $0 < f(x) < 1$
(B)	If $1 < x < 2$, then $f(x)$ satisfies	**(Q)** $f(x) < 0$
(C)	If $3 < x < 5$, then $f(x)$ satisfies	**(R)** $f(x) > 0$
(D)	If $x > 5$, then $f(x)$ satisfies	**(S)** $f(x) < 1$

(2007)

The Correct Matching is

(a) (A–P; B–Q; C–Q; D–P)
(b) (A–Q; B–P; C–Q; D–P)
(c) (A–P; B–P; C–Q; D–Q)
(d) (A–Q; B–Q; C–P; D–P)

Answer Key

CHAPTER - 5 | SETS, RELATIONS & FUNCTION

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (d)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (c)
6. (a)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (d)
11. (b)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (a)
16. (a)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (c)	**22.** (d)	**23.** (a)	**24.** (a)	**25.** (d)
26. (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (c)
31. (a)	**32.** (c)	**33.** (a)	**34.** (d)	**35.** (c)
36. (a)	**37.** (a)	**38.** (b)	**39.** (b)	**40.** (c)
41. (d)	**42.** (d)	**43.** (b)	**44.** (d)	**45.** (c)
46. (c)	**47.** (d)	**48.** (b)	**49.** (c)	**50.** (d)
51. (d)	**52.** (c)	**53.** (16)	**54.** (16)	**55.** (60)
56. (2)	**57.** (42)	**58.** (60)	**59.** (600)	**60.** (41)
61. (6)	**62.** (1)	**63.** (3)	**64.** (1)	**65.** (0)

EXERCISE - 2 : PREVIOUS YEAR JEE MAIN QUESTIONS

1. (d)	**2.** (a)	**3.** (c)	**4.** (a)	**5.** (d)
6. (d)	**7.** (38)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (a)	**13.** (b)	**14.** (d)	**15.** (d)
16. (28)	**17.** (29)	**18.** (d)	**19.** (b)	**20.** (5)
21. (c)	**22.** (b)	**23.** (832)	**24.** (b)	**25.** (256)

CHAPTER - 5 | SETS, RELATIONS & FUNCTION

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c) **2.** (b) **3.** (d) **4.** (b) **5.** (d)

6. (d) **7.** (b) **8.** (d) **9.** (d) **10.** (c)

11. (d) **12.** (c) **13.** (a) **14.** (d) **15.** (b)

16. (a) **17.** (d) **18.** (d) **19.** (d) **20.** (d)

21. (a) **22.** (c) **23.** (a) **24.** (a) **25.** (c)

26. (d) **27.** (b) **28.** (b,c) **29.** (a,b,d) **30.** (b,c)

31. (a,b,c,d) **32.** (110) **33.** (6) **34.** (4)

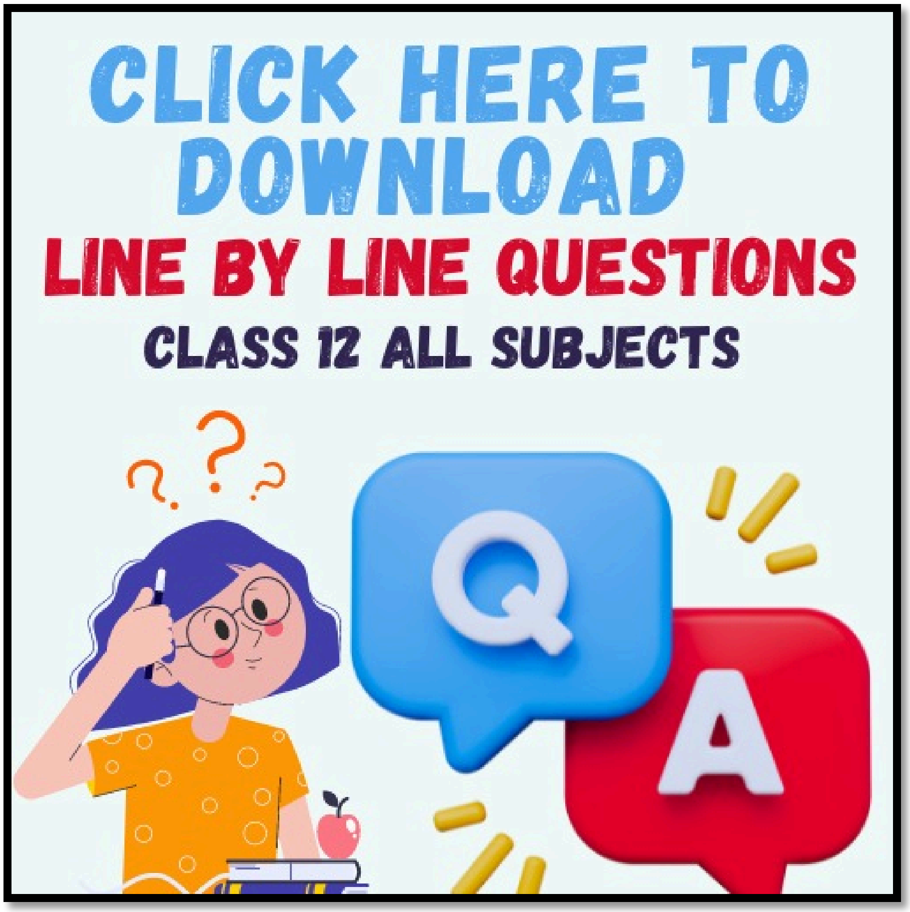
35. (a) **36.** (b) **37.** (b) **38.** (d) **39.** (b)

40. (c) **41.** (d) **42.** (d) **43.** ($x=8$)

44. ($x = 2$ or 81) **45.** ($x = 3$ or -3)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (b) **2.** (b) **3.** (c) **4.** (b) **5.** (a)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(VI TO VIII)

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Groups Rules & Regulations:

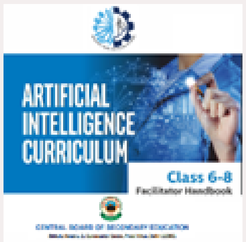
To maximize the benefits of these WhatsApp groups, follow these guidelines:

1. Share your valuable resources with the group.
2. Help your fellow educators by answering their queries.
3. Watch and engage with shared videos in the group.
4. Distribute WhatsApp group resources among your students.
5. Encourage your colleagues to join these groups.

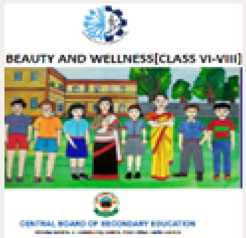
Additional notes:
1. Avoid posting messages between 9 PM and 7 AM.
2. After sharing resources with students, consider deleting outdated data if necessary.
3. It's a NO Nuisance groups, single nuisance and you will be removed.
   - No introductions.
   - No greetings or wish messages.
   - No personal chats or messages.
   - No spam. Or voice calls
   - Share and seek learning resources only.

Please only share and request learning resources. For assistance, contact the helpline via WhatsApp: +91-95208-77777.

SKILL MODULES BEING OFFERED IN MIDDLE SCHOOL

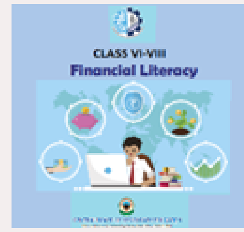


Artificial Intelligence



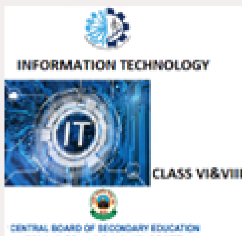
Beauty & Wellness

Design Thinking & Innovation



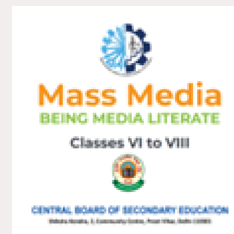
Financial Literacy

Handicrafts

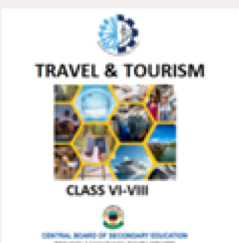


Information Technology

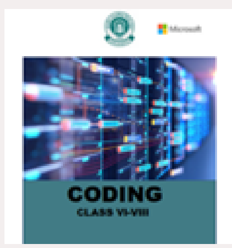
Marketing/Commercial Application



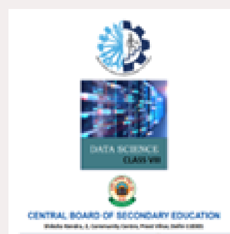
Mass Media - Being Media Literate



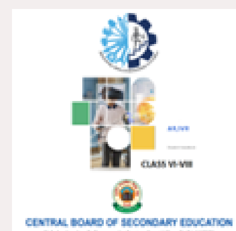
Travel & Tourism



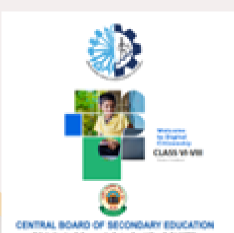
Coding



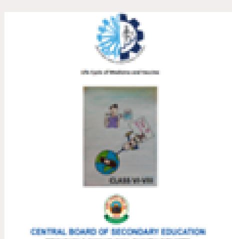
Data Science (Class VIII only)



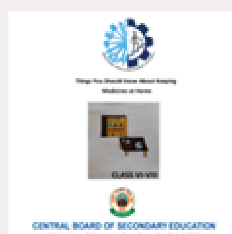
Augmented Reality / Virtual Reality



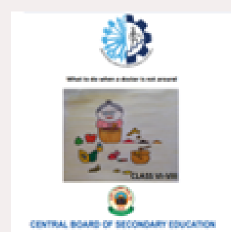
Digital Citizenship



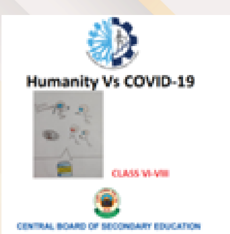
Life Cycle of Medicine & Vaccine



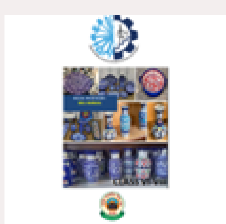
Things you should know about keeping Medicines at home



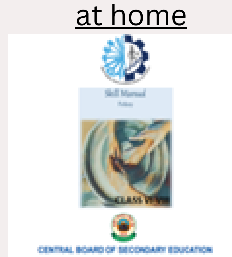
What to do when Doctor is not around



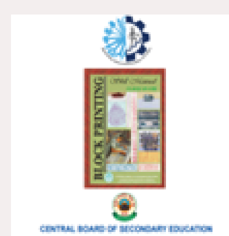
Humanity & Covid-19



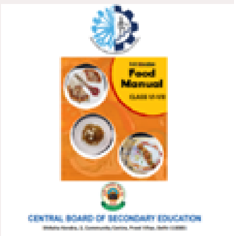
Blue Pottery



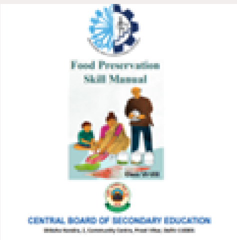
Pottery



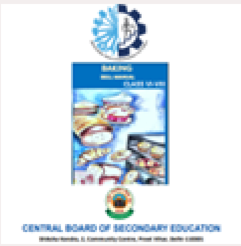
Block Printing



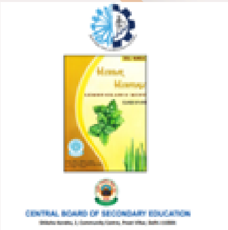
[Food]



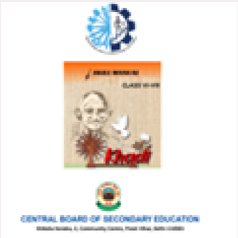
[Food Preservation]



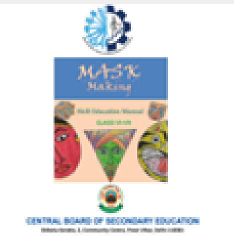
[Baking]



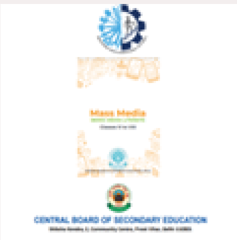
[Herbal Heritage]



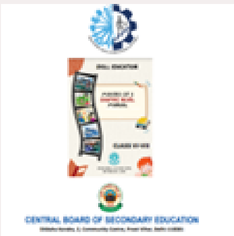
[Khadi]



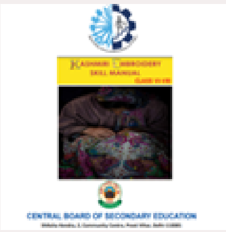
[Mask Making]



[Mass Media]

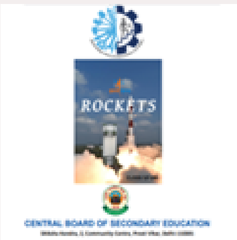


[Making of a Graphic Novel]

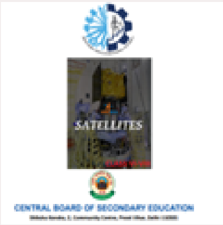


[Kashmiri Embroidery]

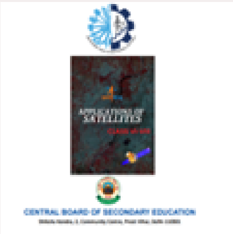
[Embroidery]



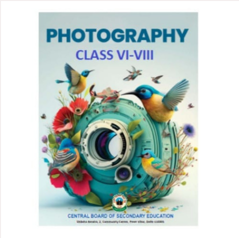
[Rockets]



[Satellites]

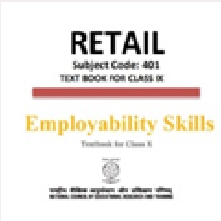


[Application of Satellites]



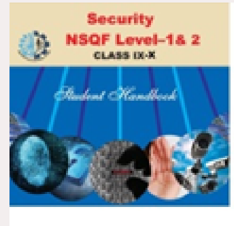
[Photography]

SKILL SUBJECTS AT SECONDARY LEVEL (CLASSES IX – X)

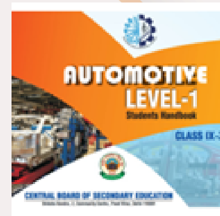


Retail

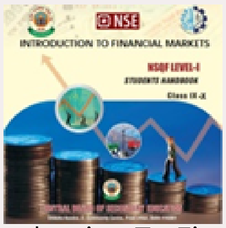
Information Technology



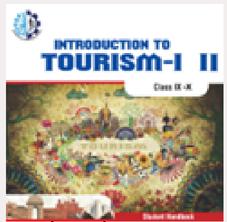
Security



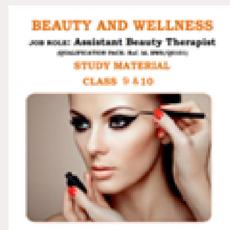
Automotive



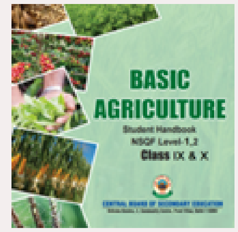
Introduction To Financial Markets



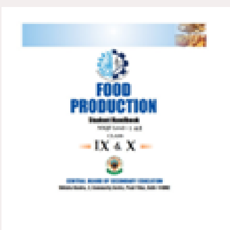
Introduction To Tourism



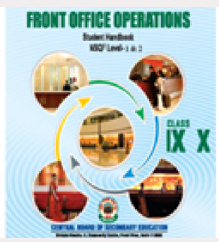
Beauty & Wellness



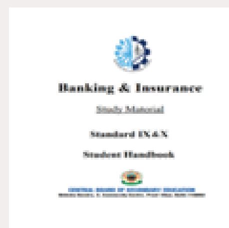
Agriculture



Food Production

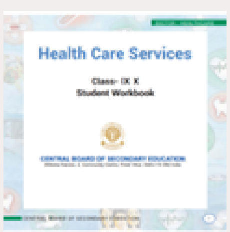


Front Office Operations



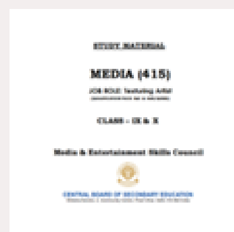
Banking & Insurance

Marketing & Sales

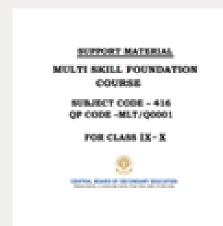


Health Care

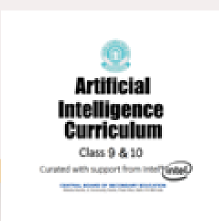
Apparel



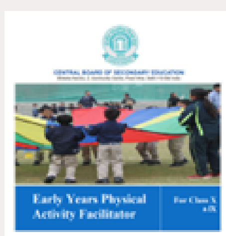
Multi Media



Multi Skill Foundation Course



Artificial Intelligence

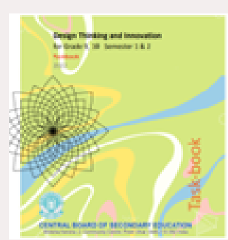


Physical Activity Trainer

Data Science

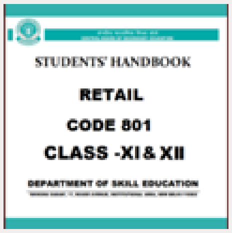
Electronics & Hardware (NEW)

Foundation Skills For Sciences (Pharmaceutical & Biotechnology)(NEW)

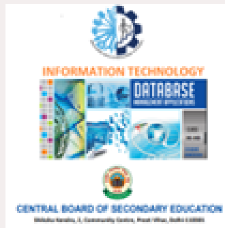


Design Thinking & Innovation (NEW)

SKILL SUBJECTS AT SR. SEC. LEVEL (CLASSES XI – XII)

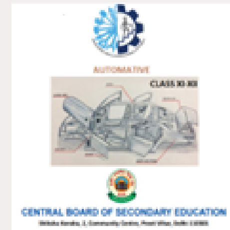


Retail



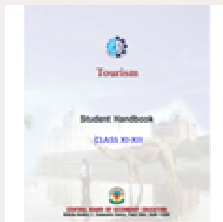
InformationTechnology

Web Application

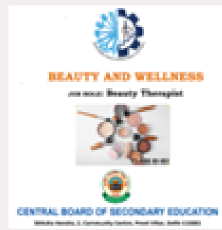


Automotive

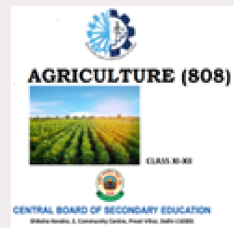
Financial Markets Management



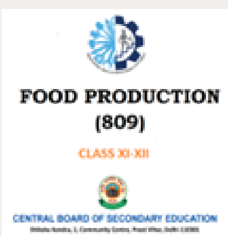
Tourism



Beauty & Wellness

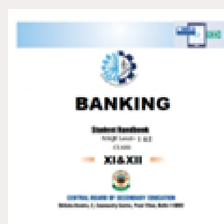


Agriculture

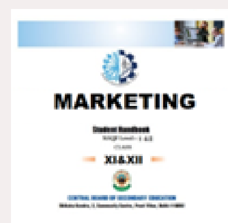


Food Production

Front Office Operations

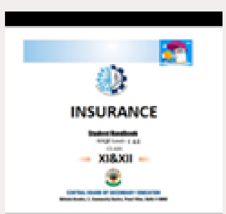


Banking

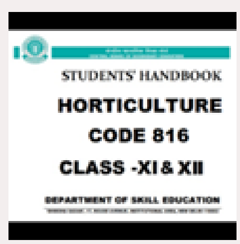


Marketing

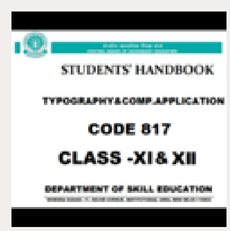
Health Care



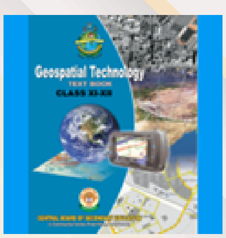
Insurance



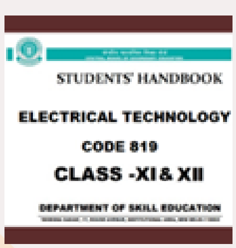
Horticulture



Typography & Comp. Application

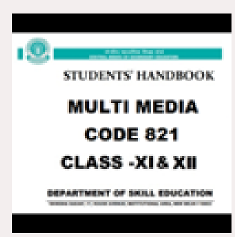


Geospatial Technology

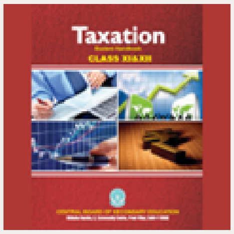


Electrical Technology

Electronic Technology

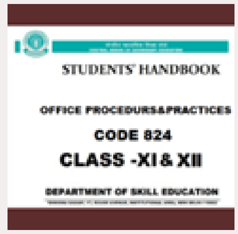


Multi-Media

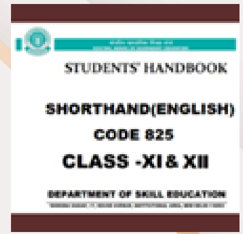


Taxation

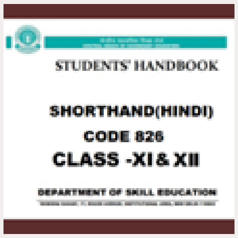
Cost Accounting



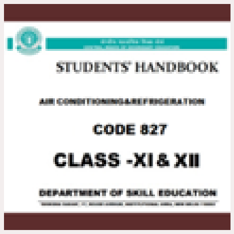
Office Procedures & Practices



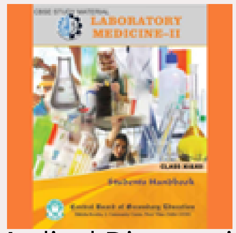
Shorthand (English)



Shorthand (Hindi)

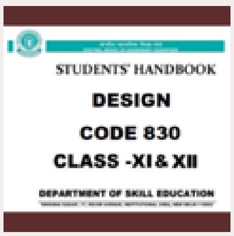


Air-Conditioning & Refrigeration

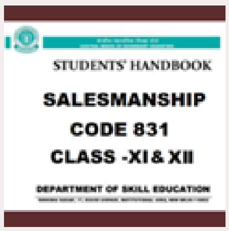


Medical Diagnostics

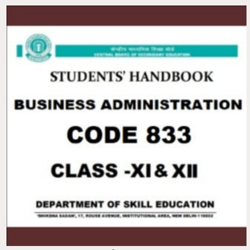
Textile Design



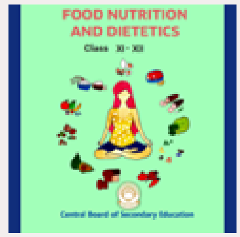
Design



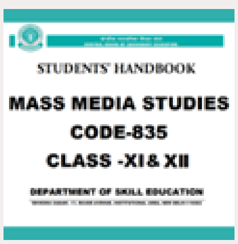
Salesmanship



Business Administration

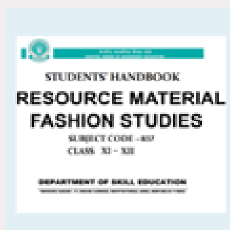


Food Nutrition & Dietetics

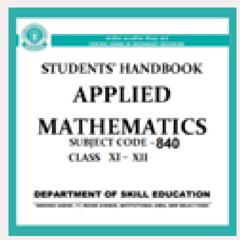


Mass Media Studies

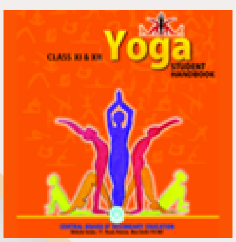
Library & Information Science



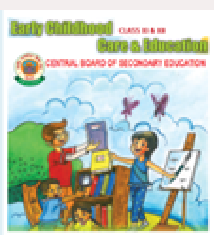
Fashion Studies



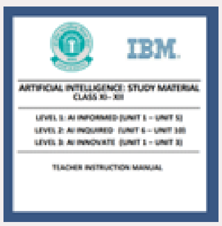
Applied Mathematics



Yoga

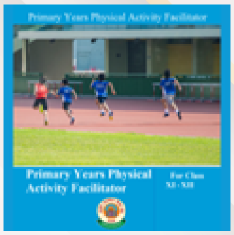


Early Childhood Care & Education

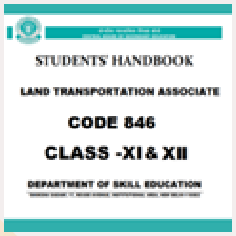


Artificial Intelligence

Data Science



Physical Activity Trainer(new)



Land Transportation Associate (NEW)

Electronics & Hardware (NEW)

Design Thinking & Innovation (NEW)